Linear Quadratic Regulator Design for Wheelchair Control Using Monte-Carlo Simulation Method

Efgan UĞUR[1], Tolgay KARA[1], and Abdulhafez ABDULHAFEZ[2]

[1]Gaziantep University, Engineering Faculty, Department of Electrical and Electronics Engineering, Gaziantep, TURKEY
efganugur@gantep.edu.tr, kara@gantep.edu.tr
[2]Hasan Kalyoncu University, Engineering Faculty, Department of Software Engineering, Gaziantep, TURKEY
abdul.hafez@hku.edu.tr

Abstract

In this paper, we present Monte-Carlo simulation for a linearized wheelchair model that use Linear Quadratic Controller. The controller has two weighting matrices which are Q and R. These weighting matrices are important to select optimal gain matrix value. Trial and Error method has been usually used in Linear Quadratic Controller because of simplicity but very difficult to choose the best values that have good control performances. Without giving up the simplicity of the Trial and Error method, we propose Monte-Carlo method. So, many closed loop simulations using Monte-Carlo method, optimal gain matrix value is calculated. Effectiveness of the both method is discussed comparatively. The results indicate that the Monte-Carlo method are more powerful to select optimal gain matrix and better performance for the stability of the system.

1. Introduction

Most of the elderly people have difficulties in using manuel wheelchairs due to partial paralysis and tremors. The electric wheelchair can provide a solution, but it is not appropriate for older adults with a cognitive impairment, such as dementia, as these individuals do not have the cognitive capacity required to effectively and safely maneuver the wheelchair. Also, it is very difficult for this type of disabled people to control the electric wheelchair with a joy stick. Assistive technologies and robotic wheelchairs have been developed to provide reliable and safe environment for this population [1].

Many studies in literature focus on robotic wheelchairs, among which are fuzzy controller based robotic wheelchairs [2, 3], studies that use PID controller algorithms [4, 5], and many more. Combination of multiple control methods is also employed for the electric wheelchair control such as fuzzy-PID controllers [6, 7]. Although many researches have been carried out on controller design for the wheelchair, Linear Quadratic Regulator (LQR) design using Monte-Carlo simulation method has not been applied to this problem.

LQR control design using Monte-Carlo simulation is reported on the pendulum model in literature. Results of the mentioned study proves that LQR controller technique using Monte-Carlo simulation method has advanced outcomes for stability of system [8]. So, we propose to apply the Monte-Carlo simulation method with LQR controller on our linearized wheelchair model.

Most of the time, the design parameters of LQR are chosen by Trial and Error method and depend a lot on the designer's experience [9, 10]. But it is often cumbersome and tedious to tune the controller gains via Trial and Error method. There are some attempts such as Bryson rule and Genetic Algorithm in literature, but they have big computational load [11, 12]. So, we propose Monte-Carlo simulation as a more powerful method compared to the Trial and Error, which improves performance without giving up the simplicity.

The paper is organized as follows. Section 2 presents the linearized model of the wheelchair. Section 3 includes controller design and Monte-Carlo simulation method. Simulation model and results are presented in Section 4. Ultimately, conclusion and discussion is given in Section 5.

2. Linearized Model of Wheelchair

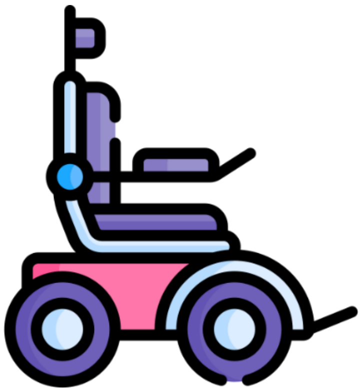

Fig. 1. Electric wheelchair

A simple diagram of the electric wheelchair can be seen in Fig. 1. Electric wheelchair moves with electric power rather than manual power. The wheelchair is operated via a joy-stick interface in most of the electric wheelchairs. In today's technology, there are robotic wheelchairs that use assistive technologies instead of the joy-stick interface. Dynamically robotic wheelchair model is similar to that of the electric wheelchair. They only differ in the interface components like camera, sensor, touch screen, etc. The parameters that effect the dynamics of wheelchair motion like the chassis of the wheelchair, user mass, DC motor torque are common in robotic wheelchair and standard electric wheelchair. Thus, the dynamic model of the electric wheelchair can be used for robotic wheelchair model as well. Selecting the proper state variables and system parameters is a key issue in modeling. Here, we present vision based robotic wheelchair model and we use state variables of displacement x, velocity $\dot{x}$, wheelchair directional angle θ, and its derivative $\dot{\theta}$ in respective order.

Dynamic wheelchair linearized state space model is given below [13]. The linearized wheelchair model can be arranged in state space form as Equation (1)

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases} \tag{1}$$

where

$$x = \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix}, \quad u = [V_a]$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{2k_m k_e a}{R r_w^2 b} & -\frac{m_p^2 g l^2}{b} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \frac{2k_m k_e}{R r_w I_T}\left(1 + \frac{m_p l a}{r_w b}\right) & \frac{m_p^3 g l^3}{I_T b} & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ \frac{2k_m a}{R r_w b} \\ -\frac{2k_m}{R I_T}\left(1 + \frac{m_p l a}{r_w b}\right) \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

3. Controller Design

3.1. LQR Design with Trial and Error

As a state-feedback controller LQR algorithm is applied on the wheelchair model. A state feedback law that minimized the cost of functional given in Equation (2) is found by the optimal control approach.

$$J = \int_0^\infty (x^T Q x + U^R R U)\, dx \tag{2}$$

One of the weighting matrices Q is used to penalize bad performance and the other weighting matrice R is used to penalize actuator effort. Their values calculated using Trial and Error method are assigned as given below.

$$\begin{cases} Q = \begin{bmatrix} 0.1 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix} \\ R = [1] \end{cases} \tag{3}$$

As a result, the state feedback gain matrix K_{TRE} that minimizes the cost using Trial and Error method is calculated as follows.

$$K_{TRE} = [3.1623 \quad 20.4120 \quad -142.3382 \quad 0.7952] \tag{4}$$

3.2. LQR Design with Monte-Carlo Simulation Method

Monte-Carlo simulation is a computational technique that uses repeated random samples of variables to analyze behavior of the system. In our study, Monte-Carlo simulation is tested on the diagonal constants of Q and R matrices, represented by $c1$, $c2$, $c3$, $c4$ and $c5$. Range of c values are between in 1-100.

$$\begin{cases} Q = \begin{bmatrix} c1 & 0 & 0 & 0 \\ 0 & c2 & 0 & 0 \\ 0 & 0 & c3 & 0 \\ 0 & 0 & 0 & c4 \end{bmatrix} \\ R = [c5] \end{cases} \tag{5}$$

Monte-Carlo simulation outcome is obtained by using randomly 1000 iteration of the given diagonal constants. Using the Monte-Carlo simulation results, optimal value of the gain matrix for Monte-Carlo simulation method is calculated.

The linearized state-space model has four state variables (x, $\dot{x}$, θ, $\dot{\theta}$), so the feedback gain matrix K_{MC} is obtained by Algebraic Riccati equation [9] and is designed to have four control parameter values as its elements. Fig. 2 shows iterations for the first element of the gain matrix. These iterations are applied for 1000 values. Blue circles indicate the values of gain matrix and red line points out the mean of the elements of the gain matrix. Yellow line represents the standard deviation of the elements of the gain matrix. The same iteration outcomes for the other three elements of the feedback gain matrix are similarly presented in Figs. 4,6 and 8, respectively representing the second, third and fourth elements.

In Fig. 3, we present the distribution of the 1000 iterations and their intensified values for the first control parameter. It is observed from the inspection of Fig. 3 that 480 values out of 1000 lie in the interval of 0.9747. Fig. 5 shows that 430 values out of 1000 are in the interval of 11.2725 for the second control parameter. Similarly, for the third control parameter Fig. 7 reveals that -143 is the most intensified value as 650 out of 1000 values. Finally, 960 values out of 1000 are in the 0.8637 interval as shown in Fig. 9 for the fourth control parameter. Consequently, the most appropriate control parameters in the feedback gain matrix are determined through the Monte-Carlo simulation method and the gain matrix is determined as follows:

$$K_{MC} = [0.9747 \quad 11.2725 \quad -143 \quad 0.8637] \tag{6}$$

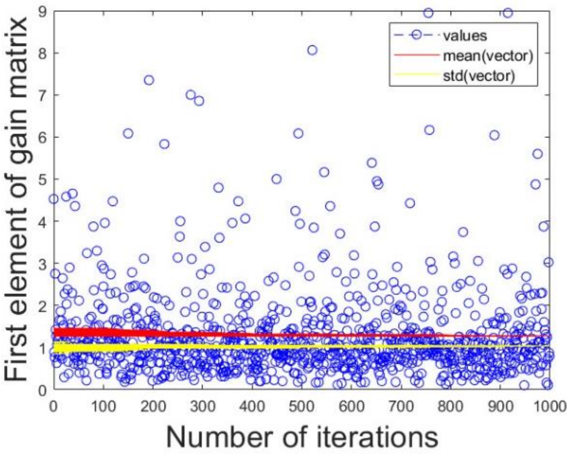

Fig. 2. Iterations for first element of optimal gain matrix

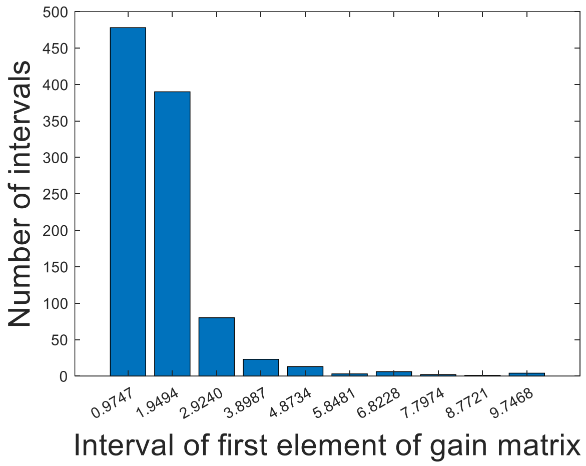

Fig. 3. Iterations for first element of optimal gain matrix (bar chart)

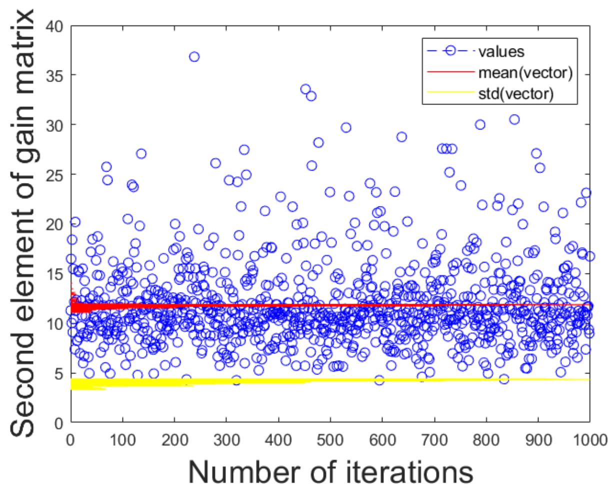

Fig. 4. Iterations for second element of optimal gain matrix

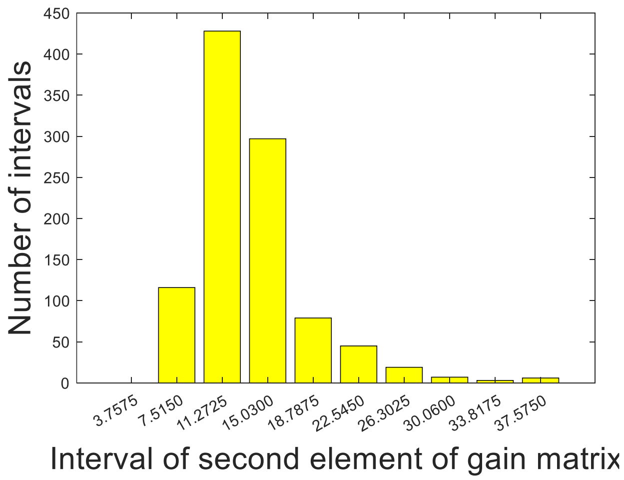

Fig. 5. Iterations for second element of optimal gain matrix (bar chart)

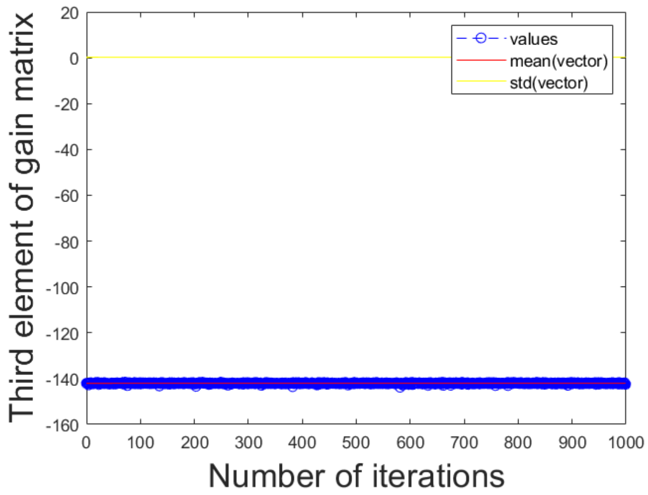

Fig. 6. Iterations for third element of optimal gain matrix

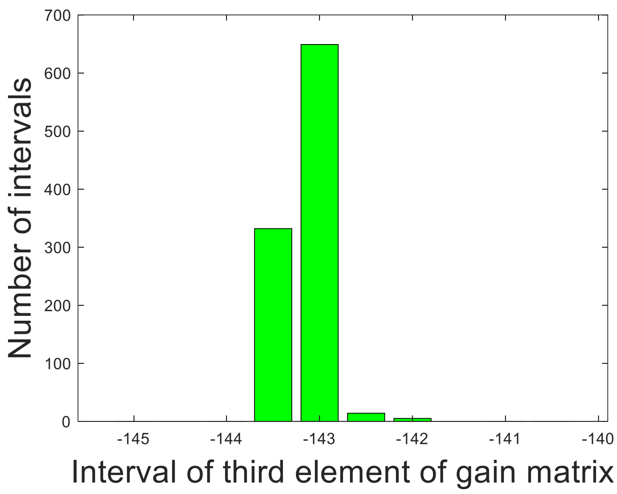

Fig. 7. Iterations for third element of optimal gain matrix (bar chart)

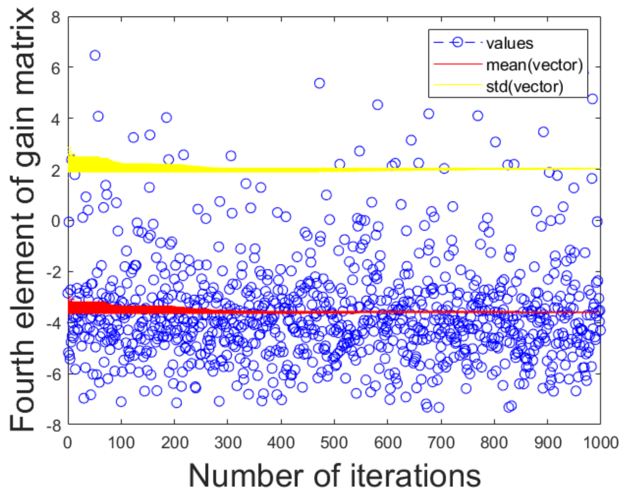

Fig. 8. Iterations for fourth element of optimal gain matrix

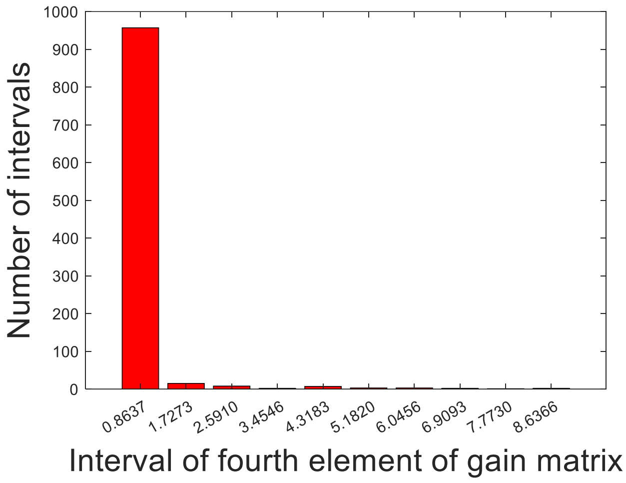

Fig. 9. Iterations for fourth element of optimal gain matrix (bar chart)

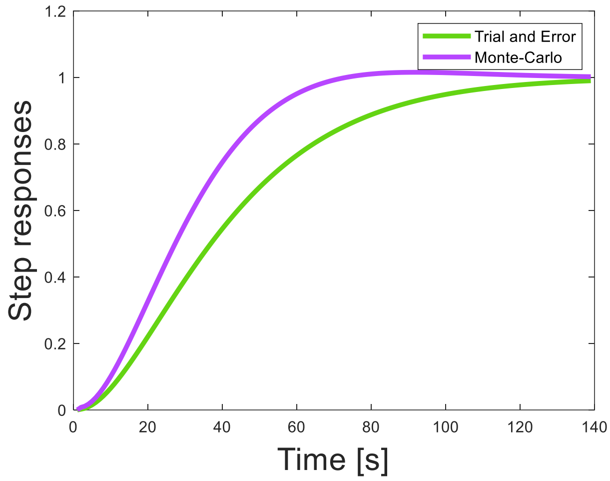

Fig. 10. Step responses of Monte-Carlo simulation and Trial and Error methods

4. Simulation Model and Results

The simulations are implemented using the linearized wheelchair model provided in Section II in state-space form with A, B, C and D matrices. Model parameters used in simulations and their values are given in Table 1.

Table 1. Parameter of wheelchair

Parameter	Description	Value	Units
g	Gravitation	9.81	m/s^2
r_w	Wheel radius	0.1778	m
m_w	Wheel mass	2.8	kg
m_p	Body mass with load	75.4	kg
I_w	Wheel inertia	0.03	kgm^2
I_p	Body inertia	0.44	kgm^2
l	Distance from body's COG	0.52	m
k_m	Motor torque	0.75	Nm/A
k_e	Back EMF	0.75	V/(rad/s)
R	Terminal Resistance	2.38	Ohms

Three simulations are carried out and results are presented with the purpose of comparing the Trial and Error and Monte-Carlo simulation methods for LQR design. Firstly, step responses of the system under the two methods for state variable θ are shown in Fig. 10. It is clearly revealed by the simulation responses in Fig. 10 that the feedback system designed using the Monte-Carlo simulation method reaches the desired value in significantly shorter time. Secondly, tracking performances of both methods for state variable θ are analyzed. Fig. 11 shows that Trial and Error is weaker in tracking the square wave reference path. Purple line that represents the Monte-Carlo method response is able to track the black reference line with better accuracy. Green line represents the Trial and Error method response and it is visible especially in the second cycle of the square wave test that this method is not effective as Monte-Carlo simulation in tracking the square wave reference. Additionally, error index values are calculated to validate the performances of the two methods numerically. Indices of Mean Squared Error (MSE), Integral Absolute Error (IAE) and Integral Time Absolute Error (ITAE) are calcuated and presented in Table 2. The error index values show that Monte-Carlo simualtion method yields significantly better performance than Trial and Error method.

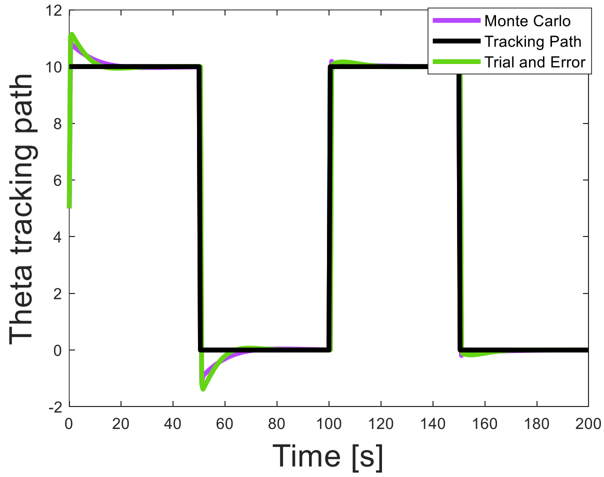

Fig. 11. Theta tracking path for Monte-Carlo simulation and Trial and Error methods

Lastly, a square wave test signal is applied to the system as a disturbance input, and attenuation of disturbance for both methods is examined. Fig. 12 shows the resuts of disturbance attenuation tests for Monte-Carlo simulation and Trial and Error methods. Also, error index values of the disturbance attenuation test results are presented in Table 3. It is clear that Monte-Carlo simulation method has better performance in terms of IAE and ITAE over Trial and Error method. However, the MSE error index is larger for the Monte-Carlo simulation response in distubance attenuation test. It should be noted here that MSE provides a measure of average error in performance, while IAE and ITAE represent measures of accumulation of error in time, with an additional emphasis on the response speed in the latter. Consequently, overall performance of Monte-Carlo simulation method in disturbance attenuation is more satisfactory than Trial and Error method especially when error accumulation over time is considered.

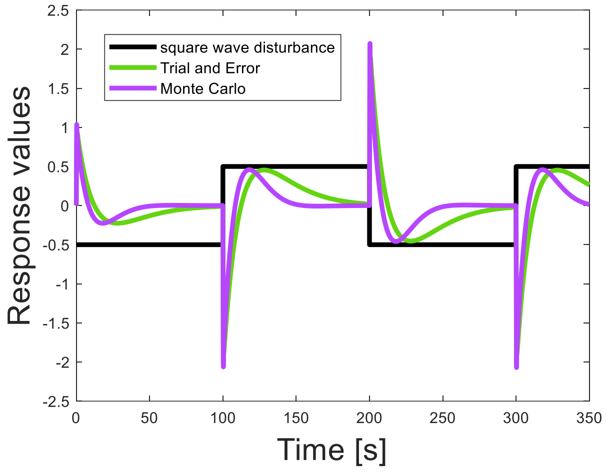

Fig. 12. Disturbance attenuation tests for Monte-Carlo simulation and Trial and Error methods

Table 2. Error index values for Fig. 11

Reference Wave	Design Method	MSE	IAE	ITAE
Square	Monte-Carlo	0.0004	15.84	1630
	Trial and Error	0.0005	16.49	1683

Table 3. Error index values for Fig. 12

Disturbance	Design Method	MSE	IAE	ITAE
Square	Monte-Carlo	0.027	351.7	6.10×10^4
	Trial and Error	0.015	355.5	6.18×10^4

5. Conclusions

This paper presents the results of the study on LQR design for electric wheelchair control using Monte-Carlo simulation method. Optimal feedback gain matrix in LQR depends on the selection of Q and R matrices, which plays an important role to control the wheelchair efficiently. This task is achieved by Monte-Carlo simulation method with 1000 iterations in the study. Monte-Carlo simulation method is compared with Trial and Error, which is commonly used in similar LQR design problems. The comparison is realized on a computer simulation platform using linearized wheelchair model for step response, trajectory tracking and disturbance attenuation tests. The test outcomes reveal that Monte-Carlo simulation method has some improvement in performance compared to Trial and Error method. Overall, the results indicate that Monte-Carlo simulation proves to be an important tool for finding the optimal feedback gain matrix in LQR control of electric wheelchairs, with enhanced reference tracking and disturbance attenuation performances. The results provide important insights into wheelchair control problem, and contributes to developing new technologies for safety and comfort of disabled people.

6. Acknowledgement

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