Solution to Short Term Hydrothermal Scheduling Problem by Modified Subgradient Algorithm Based on Feasible Values

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Abstract

In this paper, the modified subgradient algorithm based on feasible values (F-MSG) is applied to a short term hydrothermal coordination problem. Power system transmission loss is inserted into this model as equality constraints via load flow equations. Unit generation constraints, transmission line capacity constraints, bus voltage magnitude constraints, off nominal tap ratio constraints, SVAR system susceptances constraints, hydraulic units' minimum and maximum reservoir volume constraints, hydraulic units' starting and final reservoir volume constraints are also added into the optimization problem. Since all constraints in the nonlinear programming model are functions of complex bus voltages, off-nominal tap ratios and SVAR system susceptances (once there are off nominal tap changing transformers and SVAR systems in the power system), they are taken as independent variables. The proposed approach is tested on 16 bus test system where there are four serial-parallel hydraulically coupled hydro plants. The same dispatch problem was also solved by other dispatch techniques that use pseudo spot price algorithm and classical gradient method previously. Results obtained from those algorithms and F-MSG are also compared.

1. Introduction

Short term hydrothermal coordination problem (STHCP) in power systems is a constrained non-linear optimization problem. The solution of STHCP gives all thermal and hydraulic active generations that minimize the total thermal generation cost during the operation period which can range from one day to a week. At the solution point, all possible hydraulic and electric constraints are also satisfied. During the operation period, system load values and the units, which will supply those loads, are assumed to be known. The total operation period is divided into subintervals where the system load values are assumed to remain constant.

In the literature, many methods have been developed and applied to solve STHCP. Some of these methods use the particle swarm optimization technique [1], the differential evolution based methods [2], the artificial bee colony algorithm [3], the benders decompositions based methods [4], the genetic algorithm [5] and the pseudo spot price algorithm [6]. In addition to these, a survey on the optimization methods applied to solve the STHCP can be found in [7].

The modified subgradient algorithm operating on feasible values (F-MSG) was recently developed by Kasimbeyli et al [8]. It is a deterministic solution method, which uses deterministic equations at one point to produce the next solution point being closer to the optimum solution in the solution space. In the F-MSG algorithm, the upper bound for the cost function value is specified in advance and the algorithm tries to find a solution where the cost function is less than or equal to the upper bound and all constraints are satisfied. If it finds it (feasible total cost), the upper bound is decreased a certain amount, otherwise (infeasible total cost) the upper bound is increased a certain amount. The amount of decrease or increase on the upper bound for the next iteration depends on if any feasible or infeasible total cost value was obtained in the previous iterations. This process continues until absolute value of the change in the upper bound is less than a predefined tolerance value.

F-MSG algorithm has already applied to non-convex economic dispatch problem [9-10]. Furthermore, power dispatch problem including limited energy supply thermal units [11-12] and non-convex pumped-storage hydraulic unit scheduling problem [13-14] were solved via F-MSG method. To our knowledge, the proposed algorithm has not been applied to the problem considered in this paper so far.

2. Problem Formulation

In this section, a nonlinear programming model is presented for the economic power dispatch problem considered in this paper.

$$\min F_T = \sum_{j=1}^{j_{max}} \sum_{i \in \mathbf{N}_T} F_i(P_{Gi,j}) t_j \tag{1}$$

Subject to

$$P_{Gi,j} - P_{Load\,i,j} - \sum_{k \in \mathbf{N}_{Bi}} p_{ik,j} = 0, \ i = 1,2,\cdots,N, \ j = 1,2,\cdots,j_{max} \tag{2}$$

$$Q_{Gi,j} - Q_{Load\,i,j} - \sum_{k \in \mathbf{N}_{Bi}} q_{ik,j} = 0, \ i = 1,2,\cdots,N, \ j = 1,2,\cdots,j_{max} \tag{3}$$

$$P_{Gi}^{min} \le P_{Gi,j} \le P_{Gi}^{max}, \qquad i \in \{\mathbf{N}_T \bigcup \mathbf{N}_H\}, \ j = 1,2,\cdots,j_{max} \tag{4}$$

$$Q_{Gi}^{min} \le Q_{Gi,j} \le Q_{Gi}^{max}, \qquad i \in \{\mathbf{N}_T \bigcup \mathbf{N}_H\}, \ j = 1,2,\cdots,j_{max} \tag{5}$$

$$p_{l,j} \le p_l^{max}, \qquad l \in \mathbf{L}, \ j = 1,2,\cdots,j_{max} \tag{6}$$

$$U_i^{min} \le U_{i,j} \le U_i^{max}, i = 1,2,\cdots,N, i \ne ref, vc, j = 1,\cdots,j_{max} \tag{7}$$

$$V_i^{min} \le V_{i,j} \le V_i^{max}, \qquad i \in \mathbf{N}_H, \quad j = 1,\cdots,j_{max} \tag{8}$$

$$V_{i,j} = V_{i,j-1} - q_{GH}(P_{Gi,j}) t_j, \quad i \in \mathbf{N}_H \ j = 1,\cdots,j_{max} \tag{9}$$

$$q_{Hi}^{min} \le q_{GH}(P_{Gi,j}) \le q_{Hi}^{max}, \quad i \in \mathbf{N}_H, j = 1,\cdots,j_{max} \tag{10}$$

$$V_{i,0} = V_i^{start}, \quad i \in \mathbf{N}_H \tag{11}$$

$$V_{i,j\,max} = V_i^{end}, \quad i \in \mathbf{N}_H \tag{12}$$

The meanings of the symbols used in this paper are given in the List of Symbols section.

2.1. Determination of Line Flows and Power Generations

In order to express the total cost rate function in terms of independent variables of our optimization model, line flows should be written in terms of bus voltage magnitudes and phase angles, off-nominal tap settings, susceptance values of svar systems (see equations (1), (2) and (3)).The necessary equations, giving the active and reactive power flows ($p_{ik,j}$ and $q_{ik,j}$) over the line that is connected between buses i and k in the jth subinterval in terms of the independent variables, can be found in reference [9]. Using those equations and equation (2), the active and reactive power generations of the ith unit connected to bus i in the jth subinterval can be calculated as below:

$$P_{Gi,j} = P_{Load\,i,j} + \sum_{j \in N_{Bi}} p_{ik,j} \quad \text{and} \quad Q_{Gi,j} = Q_{Load\,i,j} + \sum_{k \in N_{Bi}} q_{ik,j} \qquad (13)$$

Also, the total loss of the network can be calculated as;

$$p_{loss\,ik,j} = p_{ik,j} + p_{ki,j} \quad \text{and} \quad P_{LOSS} = \sum_{i \in N}\sum_{k \in N, k \neq i} p_{ik,j} \qquad (14)$$

2.2. Converting Inequality Constraints into Equality Constraints

Since the F-MSG algorithm requires that all constraints need to be expressed in equality constraint form, the inequality constraints in the optimization model should be converted into the corresponding equality constraints. The following method is used for this purpose, since it does not add any extra independent variable into the optimization model in the conversion process [9]. The double sided inequality $x_i^- < x_{i,j} < x_i^+$ in the jth subinterval can be written as the following two inequalities

$$h_i^+(x_{i,j}) = (x_{i,j} - x_i^+) \leq 0 \quad \text{and} \quad h_i^-(x_{i,j}) = (x_i^- - x_{i,j}) \leq 0 \qquad (15)$$

Then, we can rewrite the above inequalities as continuous equality forms by the following, respectively:

$$h_i^{eq+}(x_{i,j}) = \max\{0, (x_{i,j} - x_i^+)\},$$
$$h_i^{eq-}(x_{i,j}) = \max\{0, (x_i^- - x_{i,j})\} \qquad (16)$$

If $x_i^- \leq x_{i,j} \leq x_i^+$, it is obvious that $(x_{i,j} - x_i^+) \leq 0$, $(x_i^- - x_{i,j}) \leq 0$ and $\max\{0, (x_{i,j} - x_i^+)\} = 0$, $\max\{0, (x_i^- - x_{i,j})\} = 0$. So the inequality constraints in (15) can be represented by the corresponding equality constraints in (16). In this paper the inequality constraints, given in section 2, are converted into the corresponding equality constraints in this manner.

3. The Modified Subgradient Algorithm Based on Feasible Values

The independent (decision) variables of the method are made up voltage magnitudes and phase angles of the buses (except reference bus,), the tap settings of the off-nominal tap ratio transformers and the susceptance values of the SVAR systems in the network. The method uses an augmented LaGrange function that is called as sharp LaGrange function. The F-MSG algorithm proposed to solve the dispatch problem of each subinterval based on the modified subgradient method based on feasible values is given in reference [9] in detailed manner. Due to lack of space in this paper, the F-MSG

algorithm cannot be given here. The reader should refer to reference [9] to examine the F-MSG algorithm.

3.2. The Proposed Solution Technique for the STHCP

Since short term hydrothermal coordination problem (STHCP) is a large and complex optimization problem, F-MSG algorithm can be applied in each subinterval of the considered problem one by one in order to reduce the solution time and complexity of the problem. But in this case, the net water usage constraints of the hydraulic units cannot be controlled during the solution process. Therefore the following iterative method is proposed to solve STHCP described in this paper.

initial step
- Take ITER=0
- Apply F-MSG to STHCP in each subinterval one by one without considering any hydraulic constraints given by equations (8)-(12).
- At the solution point, calculate the followings and go to step 1.

$$F_{OPT}^{*(ITER)} = \sum_{j=1}^{j_{max}}\sum_{i \in N_T} F_i(P_{Gi,j})t_j , \quad P_{LOSS,j}^* = P_{LOSS,j} , \quad j = 1,2,...,j_{max}$$

Step-1
- ITER=ITER+1
- Solve the following sub-problem by using $F_{OPT}^{*(ITER-1)}$ and $P_{LOSS,j}^*$ values obtained in the previous step

$$\min \left\{\left(\sum_{j=1}^{j_{max}}\sum_{i \in N_T} F_i(P_{Gi,j})t_j\right) - F_{OPT}^{*(ITER-1)}\right\}$$

Subject to

$$\sum_{i \in N_T} P_{Gi} + \sum_{k \in N_H} P_{Gk} - P_{LOSS,j}^* - P_{LOAD,j} = 0 , \quad j = 1,2,...,j_{max}$$

$$P_{Gi}^{min} \leq P_{Gi,j} \leq P_{Gi}^{max}, \quad i \in \{N_T \cup N_H\}, \ j = 1,2,\cdots,j_{max}$$

$$V_i^{min} \leq V_{i,j} \leq V_i^{max}, \quad i \in N_H, \ j = 1,\cdots,j_{max}$$

$$V_{i,j} = V_{i,j-1} - q_{GH}(P_{Gi,j})t_j, \quad i \in N_H \ \ j = 1,\cdots,j_{max}$$

$$q_{Hi}^{min} \leq q_{GH}(P_{Gi,j}) \leq q_{Hi}^{max}, \quad i \in N_H, j = 1,\cdots,j_{max}$$

$$V_{i,0} = V_i^{start}, \quad i \in N_H$$

$$V_{i,j\,max} = V_i^{end}, \quad i \in N_H$$

- At the solution point, take
$P_{Gi,j}^{**} = P_{Gi,j}, \quad i \in N_H, \ j = 1,2,\cdots,j_{max}$ and go to step 2.

Step-2
- Take $P_{Gi}^{min} = P_{Gi}^{max} = P_{Gi,j}^{**}, i \in N_H, \ j = 1,2,\cdots,j_{max}$
- Apply F-MSG to STHCP in each subinterval one by one with considering all constraints given by equations (2)-(12). At the solution point, calculate the followings

$$F_{OPT}^{*(ITER)} = \sum_{j=1}^{j_{max}}\sum_{i \in N_T} F_i(P_{Gi,j})t_j , \quad P_{LOSS,j}^* = P_{LOSS,j} , \quad j = 1,2,...,j_{max}$$

- If ITER=1, go to step 1. Else calculate $DECR = F_{OPT}^{*(ITER)} - F_{OPT}^{*(ITER-1)}$
- If $DECR < 0$, go to step 1.
- If $DECR > 0$, Stop. $F_{OPT}^{*(ITER-1)}$ is the optimal solution to STHCP.

In the initial step, the F-MSG algorithm is applied to dispatch problem of each subinterval of the considered problem one one by one *without* considering any hydraulic constraints in order to obtain the lowest possible fuel cost value of the considered system. This lowest cost value is taken as the starting point for the proposed method. It is obvious that this starting solution point is an infeasible solution to STHCP since hydraulic constraints were not considered in the initial step. Therefore the active power generations should be rescheduled in order to obtain a new solution whose total cost value is closer to starting point's total cost and also all the hydraulic constraints are satisfied.

In step 1, hydraulic unit scheduling is performed by solving the sub-problems over the whole operation period. *In this step, active power generations of the units are taken as decision variables in the F-MSG algorithm.* Although all hydraulic constraints are satisfied in this new solution point, it is not an exact solution to STHCP since in the sub-problem of step 1 any constraints related to the exact reactive power generation consumption balance, exact reactive power generation limits and security constraints (line flow and bus voltage magnitude constraints) are not considered. Therefore in order to find an exact solution to STHCP, power generations of thermal units should be rescheduled while active power generation of the hydraulic units remain constant at the values obtained in step 1.

Thermal unit scheduling is performed in step 2. This time, F-MSG can be applied in each subinterval one by one without having any violations on the hydraulic constraints since the active generations of the hydraulic units remain constant at the values obtained in step 1. Note that the voltage magnitudes and phase angles of the buses (except the reference bus), tap settings of the off-nominal tap ratio transformers and susceptance values of the svar systems in the network are taken as decision variables in step 2 [9].

Step 1 and step 2 together can be considered as an iteration of the proposed solution algorithm. In step 1, the hydraulic unit scheduling is done while optimal operation of thermal units are found in step 2 with already calculated hydraulic unit generation in step 1. It is obvious that the solutions calculated at the end of each iteration are the actual solutions to STHCP since the whole model described by equations (1)-(12) are considered in step 2. Therefore we carry out these iterations until there is no any further decrease on the optimal cost value.

4. Numeric Example

The proposed dispatch technique is tested on 16 bus test system which has nine generating units, where five of them are thermal and four of them are hydraulic units. Cost rate functions for thermal units are taken as convex functions. A 24-hr operation period having six equal subintervals is considered. Please refer to [6] for the system's single-line diagram, line, bus, load and generator data. Reservoir storage limits, starting and ending water volumes, total water amounts to be used by hydraulic units and hydraulic relationship among the hydraulic units can be also found in [6]. Active power transmission capacities for all lines are taken as 0.85 p.u. Bus 1 is chosen as the reference bus, and its voltage magnitude and phase angle are taken as 1.05 p.u. and 0.0 rad, respectively. The lower limit of voltage magnitudes for all buses, except the reference bus, is taken as 0.95 p.u. The lower and upper limits for all busses are taken as 0.90 and 1.10, respectively. The simulation program was coded in MATLAB 6.1 and GAMS 21.5 with a Conopt-type solver.

First, we applied the initial step described in section 3.2 and found the lowest possible fuel cost value of the dispatch problem as 128386.0623 R. In this step, we used the same initial generation values taken in ref [6]. The solution point data obtained in the initial step are given in table 1 and table 2, respectively.

Table 1. Active power generations of the units obtained in the initial step (all values are in pu).

	Subinterval (j)					
	1	*2*	*3*	*4*	*5*	*6*
P_{G1}	1.6858	2.4865	2.4489	2.1175	2.6043	2.2038
P_{G4}	0.4501	0.4500	0.4521	0.8582	0.5407	0.4500
P_{G5}	0.5820	0.6293	0.8510	0.9917	1.0565	0.4850
P_{G8}	0.5000	0.5000	0.5000	0.8903	0.5000	0.6899
P_{G15}	0.4500	0.4500	0.4500	0.4500	0.4500	0.4500
P_{G10}	1.1859	1.3500	1.3500	1.3447	1.3500	0.9848
P_{G12}	0.4990	0.7985	0.9472	0.9230	0.8611	0.8611
P_{G14}	0.5420	0.6939	0.7988	0.8088	0.8580	0.6456
P_{G16}	1.0472	1.1539	1.2278	1.2895	1.2206	1.0070
P_{LOSS}	0.1419	0.2121	0.2758	0.2738	0.2913	0.1770
P_{LOAD}	6.8000	8.3000	8.7500	9.4000	9.1500	7.6000

Table 2. Starting and final reservoir volume of hydraulic units obtained in the initial step (all values are in acre-ft).

Hydraulic Unit (i)	$V_{i,0}$	$V_{i,6}$
10	50000.00	41527,78
12	45000.00	49265,34
14	46600.00	38455,80
16	40000.00	54704,65

As it seen from table 1 that big portions of the load demand are supplied by the hydraulic units, which are connected to bus 10, 12, 14 and 16, in all subintervals. It is also obvious from table 2 that the final reservoir volume constraints are not satisfied. Note that the hydraulic units' final reservoir volumes are taken as V_{10}^{end}=48000, V_{12}^{end}=46600, V_{14}^{end}=40600 and V_{16}^{end}=50600 acre-ft [6].

After that, we began the first iteration by carrying out the calculations in step 1. By using the solution obtained in the initial step, we found an *intermediate* solution in which all the hydraulic constraints are satisfied. The solution point results obtained in step 1 of the first iteration are given in table 3 and table 4, respectively.

Table 3. Active power generations of the units obtained in step 1 of the first iteration (all values are in pu).

	Subinterval (j)					
	1	*2*	*3*	*4*	*5*	*6*
P_{G1}	2.5199	2.5199	2.5199	2.5199	2.5199	2.5199
P_{G4}	1.0034	1.0034	1.0034	1.0034	1.0034	1.0034
P_{G5}	0.9883	0.9883	0.9883	0.9883	0.9883	0.9883
P_{G8}	0.6217	0.6217	0.6217	0.6217	0.6217	0.6217
P_{G15}	0.4500	0.4500	0.4500	0.4500	0.4500	0.4500
P_{G10}	0.2857	0.8183	0.9533	0.7506	0.9775	1.0839
P_{G12}	0.0002	0.6202	0.4678	1.1292	1.0389	0.0985
P_{G14}	0.1386	0.3781	1.0460	1.0377	0.7078	0.0901
P_{G16}	0.9342	1.1122	0.9755	1.1730	1.1337	0.9214
P_{LOSS}	0.1419	0.2121	0.2758	0.2738	0.2913	0.1771
P_{LOAD}	6.8000	8.3000	8.7500	9.4000	9.1500	7.6000

Table 4. Starting and final reservoir volume of the hydraulic units obtained in step 1 of the first iteration (all values are given in acre-ft).

Hydraulic Unit (i)	$V_{i,0}$	$V_{i,6}$
10	50000.00	48000.00
12	45000.00	46600.00
14	46600.00	40600.00
16	40000.00	50600.00

It is seen from table 4 that all the hydraulic constraints are met in the solution point of step 1. Note that constraints related to reactive power generation consumption balance, reactive power generation limits and security constraints are not considered in sub-problem of step 1. Therefore we should apply step 2 in order to find an exact solution in which all constraints given in section 2 are satisfied. The solution point results obtained in step 2 are given in table 5 and table 6, respectively.

Table 5. Active power generations of the units obtained in step 2 of the first iteration (all the values are in pu).

	Subinterval (j)					
	1	*2*	*3*	*4*	*5*	*6*
P_{G1}	1.8894	1.9808	2.1581	2.0041	2.1768	1.9636
P_{G4}	1.1969	1.0147	0.8976	0.9064	0.8737	1.1072
P_{G5}	1.0736	1.0798	1.1814	1.2802	1.2318	1.0535
P_{G8}	0.8198	0.8886	0.8652	0.9243	0.8172	0.8919
P_{G15}	0.6036	0.5871	0.4500	0.4500	0.4500	0.5549
P_{G10}	0.2857	0.8183	0.9533	0.7506	0.9775	1.0839
P_{G12}	0.0002	0.6202	0.4678	1.1292	1.0389	0.0985
P_{G14}	0.1386	0.3781	1.0460	1.0377	0.7078	0.0901
P_{G16}	0.9342	1.1122	0.9755	1.1730	1.1337	0.9214
P_{LOSS}	0.1421	0.1798	0.2448	0.2554	0.2575	0.1650
P_{LOAD}	6.8000	8.3000	8.7500	9.4000	9.1500	7.6000

Then we carry out further iterations. Total cost values obtained at the end of each iterations are given in table 7. As it seen from table 7 that the total cost value is increased at the end of the fourth iteration from 147191.1110 R to 147250.7857 R, so we finished the proposed method. Therefore the solution obtained at the end of third iteration is the optimal solution to STHCP. The optimal solution data are given in table 8 and table 9, respectively.

In the literature, the same dispatch problem was also solved by using pseudo spot price algorithm (PSPA) and classical gradient method (GM). Total cost values obtained via the PSPA and GM are given in table 10. The proposed method gives a total fuel cost that is 229 R lower than the one supplied by the PSPA algorithm and 331 R lower than the one supplied by the GM algorithm.

Table 6. Starting and ending reservoir volume of hydraulic units obtained in step 2 of the first iteration (all values are given in acre-ft).

Hydraulic Unit (i)	$V_{i,0}$	$V_{i,6}$
10	50000.00	48000.00
12	45000.00	46600.00
14	46600.00	40600.00
16	40000.00	50600.00

Tablo 7. Total fuel cost values obtained at the end of each iteration.

Iteration No	Total Fuel Cost Values (R)
1	147295.2922
2	147284.3710
3	147191.1110
4	147250.7857

Table 8. Active power generations of the units at the optimal solution point (all the values are in pu).

	Subinterval (j)					
	1	*2*	*3*	*4*	*5*	*6*
P_{G1}	1.8865	1.9808	2.0665	2.0399	2.1227	1.9653
P_{G4}	1.1836	1.0184	0.9291	0.9174	0.9242	1.0790
P_{G5}	1.0808	1.0843	1.2183	1.2222	1.2360	1.1018
P_{G8}	0.8241	0.8756	0.8873	0.8828	0.8271	0.8689
P_{G15}	0.5808	0.6042	0.4574	0.4913	0.4500	0.5339
P_{G10}	0.3496	0.9348	1.1434	0.9622	0.6947	0.7846
P_{G12}	0.0359	0.4497	0.6117	0.9951	0.9993	0.2631
P_{G14}	0.0278	0.5266	0.6351	0.9376	0.9727	0.2985
P_{G16}	0.9743	1.0020	1.0354	1.1992	1.1786	0.8604
P_{LOSS}	0.1434	0.1763	0.2341	0.2477	0.2555	0.1555
P_{LOAD}	6.8000	8.3000	8.7500	9.4000	9.1500	7.6000

Table 9. Starting and final reservoir volumes of hydraulic units at the optimal solution point (all values are given in acre-ft).

Hydraulic Unit (i)	$V_{i,0}$	$V_{i,6}$
10	50000.00	41527,78
12	45000.00	49265,34
14	46600.00	38455,80
16	40000.00	54704,65

Table 10. Comparison of the results that are obtained by the other methods with the one found via the F-MSG

Methods		
PSPA [6]	GM [6]	F-MSG
147420.4220 (R)	147522.335 (R)	147191.1110 (R)

5. Discussion And Conclusion

In this paper, we propose a solution method based on the F-MSG algorithm to short term hydrothermal coordination problem of a lossy power system area. The dispatch technique is tested on the 16 bus test system. As it is seen from table 10 and table 3, the proposed technique gives *lower total cost value* than those of produced by the techniques given in reference [6].

Due to lack of space, solution point complex bus voltages, active/reactive line flows, reactive power generations of the units cannot be given in the paper.

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7. List of Symbols

R : a fictitious monetary unit.

F_T : total active power generation cost of the system.

$F_i(P_{Gi,j})$: active power generation cost rate value of the ith unit in the jth subinterval.

N : number of buses in the network.

N_T : set containing all buses to which thermal generation units are connected.

N_H : set containing all buses to which hydro generation units are connected.

N_{Bi} : set containing all buses which are connected to bus i *directly.*

L : set that contains all lines in the network.

j_{max} : total number of subintervals

t_j : length of time interval j.

$P_{Gi,j}$, $Q_{Gi,j}$: active and reactive power generations of the ith unit in the jth subinterval, respectively.

$P_{Load\,i,j}$, $Q_{Load\,i,j}$: active and reactive loads of the ith bus in the jth subinterval, respectively.

$p_{ik,j}$, $q_{ik,j}$: active and reactive power flows from bus i to bus k at bus i border in the jth subinterval, respectively.

P_{Gi}^{min}, P_{Gi}^{max} : lower and upper active generation limits of the ith generation unit, respectively.

Q_{Gi}^{min}, Q_{Gi}^{max} : lower and upper reactive generation limits of the ith generation unit, respectively.

$p_{l,j}$: active power flow on line l in the jth subinterval.

p_l^{max} : maximum active power transmission capacity of line l.

U_i^{min}, U_i^{max} : minimum and maximum voltage magnitude limits of the ith bus.

$U_{i,j}$: voltage magnitude of bus the i in the jth subinterval

V_i^{min}, V_i^{max} : minimum and maximum reservoir storage limits of the ith hydro generation unit, $i \in N_H$

$V_{i,j}$: stored water volume in reservoir of the ith hydro generation unit at the end of the jth subinterval, $i \in N_H$

$q_{GH}(P_{Gi,j})$: discharge rate of the ith hydro generation unit in the jth subinterval, $i \in N_H$

q_{Hi}^{min}, q_{Hi}^{max} : lower and upper discharge rate limits of the ith hydro generation unit, $i \in N_H$

V_i^{start}, V_i^{end} : specified starting and final stored water volumes in reservoir of the ith hydro generation unit, $i \in N_H$