Solving Inversion Problem for Refractivity Estimation Using Artificial Neural Networks

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Abstract

Atmospheric refractivity index is one of the most important variable that effects the propagation direction of radio waves. This means a radar or communication system can show unexpected behaviour and performance depending on this variable. Estimation of refractivity characteristics of atmosphere is possible by using radar clutter data. This method is called refractivity from clutter (RFC). RFC is a nonlinear inversion problem. In this work, Artificial Neural Networks are studied to solve inversion problem for refractivity estimation. A training data set had to be prepared to represent ducts with refractivity parameters. Fortunately, learning and generalization capability of Neural Networks (NN) is very helpful in this point, so a good mapping of refractivity parameters can be enough for solving inversion problem.

1. Introduction

There are many methods to detect the ducts, such as; ravinsonde, rocketsonde, refractometer, doppler spread radar, atmospheric prediction models, lidar etc. [1]. But most of the time, detection of existence of a duct is not enough. For example, to predict the changes in coverage of a surveillance radar, some additional information is needed such as duct type, thickness of inversion layer, base height and strength of the duct. Especially in military, duct information is very important, because properties of a duct sometimes can determine the type of tactic in a battle, secure flight height of an attack aircraft, communication possibilities between the units or necessary altitude for jamming an enemy radar.

If the properties of a duct are known, its effects can be seen by help of some softwares. In this work, a software named as PETOOL is used for this purpose. It is an open source program, designed to serve as a research and educational tool for engineers and instructors. PETOOL uses the split step based parabolic equation method (SSPE) for modelling radio wave propagation [2]. To obtain duct properties from radar clutter data, it is necessary to use inversion techniques. In this study, neural network (NN) was used for the inversion process. Finally, performance of NN inversion was analyzed by comparing with some reference and generated propagation factors.

2. Electromagnetic Ducts and Refractivity Parameters

Electromagnetic ducts are result of atmospheric index of refraction that varies with height and time. The refractive index is calculated as

$$n = \sqrt{\varepsilon_r} = c/v \qquad (1)$$

where

ε_r is the dielectric constant of atmosphere,
c is speed of light,
v is the velocity of the EM wave.

Since n is slightly different than 1 (1,00025-1,00040) around earth surface [3], it is more practical to use the scaled index of refraction N, which is called refractivity and defined as,

$$N = (n-1).10^6 = \frac{77,6p}{T} - \frac{5,6e}{T} + \frac{3,75.10^5 e}{T^2} \qquad (2)$$

where

P is the total pressure in mbar,
e is the water vapour pressure,
T is the temperature in Kelvin.

In order to examine N gradients, the index modified refractivity is used and defined as,

$$M = N + 0,157 h \qquad (3)$$

and the electromagnetic duct occurs when,

$$dM/dh < 0 \quad or \quad dN/dh < -157 \qquad (4)$$

There are three major duct types frequently encountered in the lower atmosphere; evaporation duct, surface based duct and elevated duct as seen in Fig 1. There is always a trapping layer in duct structure and ducting effects to the radio wave propagation are similar to that of the metal waveguides.
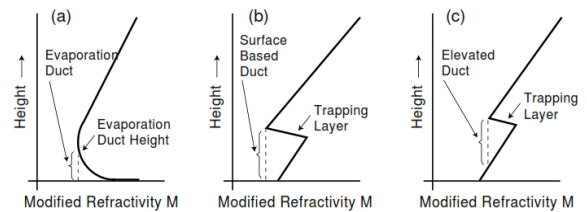

Fig. 1. Duct Types. (a) Evaporation duct, (b) surface based duct, (c) elevated duct [4].

A common refractivity structure that significantly effects the performance of air surveillance radar is surface based duct

(SBD). Typical height of an evaporation duct is between 0-30 m. Minimum required thickness of duct to trap radio waves with respect to frequency is studied by Cairns-McFeeters [5]. The relationship between duct thickness and frequency is given below.

$$f_{min} = 3,6.10^{11}.d^{-3/2} \qquad (5)$$

In the above equation f_{min} is minimum frequency and d is thickness of the duct. It can be easily calculated that the thickness of duct is 40,4 m for a 1400 MHz L band radar. On the other hand, another type of duct, elevated ducts cannot be predicted by using RFC techniques. Because the bottom of an elevated duct is not bounded by sea surface, this kind of duct does not create enough clutter to use RFC to find refractivity parameters. In this work, we limited the context of the study with only SBD parameters to focus on inversion problem.
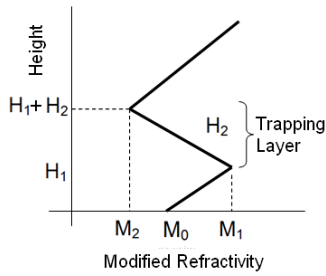

Fig. 2. Surface based duct parameters.

There are 4 main parameters to determine the characteristics of a surface based duct as shown in Fig. 2; two of them are modified refractivity parameters in M-unit and the others are height parameters in meters. Here H_2 is the thickness of trapping layer and sum of H_1 and H_2 is the total thickness of the duct.

3. Radio Wave Propagation and PETOOL Software

Radio wave propagation is affected in an inhomogeneous atmosphere by several phenomena. For example, atmospheric index of refraction changes the direction of the EM waves. Surface of earth, especially the sea surface reflects the radio wave back according to the Snell Law. A SBD on the sea surface traps the wave and the wave propagates between reflective surface and trapping layer of SBD to large distances which is not possible for normal atmospheric conditions. EM propagation problem is governed by the scalar Helmholtz equation given by,

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial z^2} + k^2 n^2 \varphi = 0 \qquad (6)$$

where $k=2\pi/\lambda$ is the free space wavenumber (λ is wavelenght), n is the refractive index, φ denotes the electric or magnetic field. Parabolic Equation (PE) model has widely been used to model the behaviour of EM wave between transmitter and receiver of a radar, over the 2-dimentional Earth's surface [6]. The PE is derived from Helmholtz equation by separating the rapidly varying phase term to obtain an amplitude function that varies slowly in range. By introducing the reduced function as $u(x,z)=exp(ikz)\varphi(x,z)$, the wave equation in terms of u is given as follows:

$$[\frac{\partial^2}{\partial x^2} + 2ik\frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial z^2} + k^2(n^2-1)]u(x,z) = 0 \qquad (7)$$

Wide-spread usage of PE has become possible after the development of Fourier Split Step Algorithm [7,8]. Split Step Parabolic Equation (SSPE) is an initial value problem starting from a reference point, and marching out in range by obtaining the field along the vertical direction at each range step through the use of step-by-step Fourier transformations. The split step solution of wide angle PE is given by [9],

$$u(x + \Delta x, z) = \exp[ik(n-1)\Delta x]$$
$$F^{-1}\left\{\exp[-ip^2\frac{\Delta x}{k}\left(\sqrt{1-\frac{p^2}{k^2}}+1\right)^{-1}]F\{u(x,z)\}\right\} \qquad (8)$$

After calculating the field inside entire computational domain by split step solutions, the propagation factor (PF), which is field strength relative to its free space value in dB, is computed by,

$$PF = 20\log|u| + 10\log x + 10\log \lambda \qquad (9)$$

In this study, PETOOL software is used to implement EM propagation and to analyze the effects of SBDs on air surveillance radars. PETOOL is introduced for the analysis and visualization of radio wave propagation through homogeneous and inhomogeneous atmosphere, by incorporating variable terrain effects with the aid of two way split step algorithm employing wide-angle propagator [2].
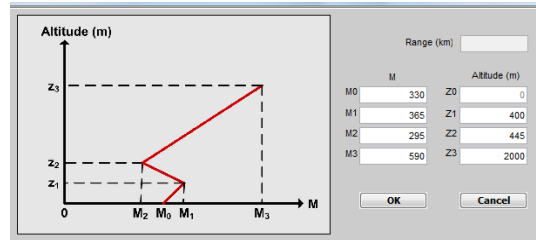

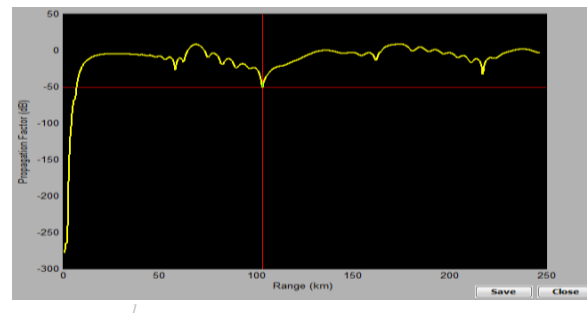**Fig. 3.** PETOOL's input panel for SBDs.

Fig. 4. PETOOL results can be seen on the order of PF.

PETOOL is developed as a research and educational tool for engineers and instructors to investigate phenomenon, such as electromagnetic ducts. Fig. 3 shows the PETOOL's input panel for SBDs. PETOOL paves the way for the user to see results of

the analysis on the order of PF, as shown in Fig. 4. All results, obtained from the analysis, can be taken from MATLAB Workspace to use for another study.

4. Methods for Solving Inversion Problem and NNs

Real PF values can be computed from real radar data using radar clutter equation and clutter models. So the problem reduces to method which enables to get modified refractivity from PF. But this is not an easy problem. Karimian summarized and analyzed inversion problems in RFC method in [10]. If the clutter cross section for a sea surface clutter is $\sigma = A_c \sigma_0$, where σ_0 is clutter cross section per unit area and A_c is the area of the radar cell, received power P_c from the clutter at the range r is computed as follows [2]:

$$P_c(r) = \frac{P_t G^2 \lambda^2 \sigma F^4(r,m)}{(4\pi^3) r^4 L} \qquad (10)$$

where P_t is the known transmitter power, G is the known antenna gain, λ is the known wave length, L is the known total assumed system loss, **F** is the PF and **m** is refractivity structure in a maritime environment. Observed power from clutter depends on $\mathbf{F^4}$ as seen in equation (10). PF depends on environmental refractivity profile through which the wave is propagated. Simulated clutter power of each candidate refractivity profile is computed and an objective function quantifies the observed clutter power and simulated power.

Simulated power can be produced by using software programs such as PETOOL. There is enough information in [2] about such programs and the methods they used. PETOOL is such a program where radar parameters and other necessary variables can be included to compute PF. But, there are an infinite number of possible duct forms and it is not feasible to work with every possible refractive profile. PETOOL uses very complex propagation formulations as PE. So, inversion problem to find out the refractivity profile from the PF that is produced by PETOOL is definitely a nonlinear problem.

5. Composition of Refractivity Parameters for NN's Training Data Set

NN is a popular algorithm that is used extensively in many different scientific and industrial areas to solve complex problems. A neural network is a massively parallel distributed structure that consists of nodes or neurons and connections whose weights are determined by training or learning [11-14].

In this paper we aimed to search the usage of NN in RFC problem solving. We prepared a set of samples from refractivity parameters and produced a set of samples from PF that corresponds to prepared refractivity parameter set by using PETOOL Software.

For inversion process, we prepared two training data sets. To represent a SBD it is enough to use 4 parameters; M_1, M_2, H_1 and H_2, as indicated in Fig 2. Parameter map of surface based duct is arranged parallel to [4], where thickness of trapping layer and M deficit is determined in Table 1. Height parameters are shown in Table 2.

Table 1. Modified refractivity parameter interval of SBD.

Parameter	Lower Bound	Upper Bound
M values (M deficit is 100)	280	380

Table 2. Height parameters of SBD

Parameters										
H_1	10	20	30	40	50	60	70	80	90	100
H_2	42	84	127	170	254	339	424	632	912	1344

To establish data training set totally 707 different combination of M_1, M_2, H_1 and H_2 are studied. This 707x4 matrix is also the target data set of NN for learning process. PETOOL Software processed this 707x4 matrix of SBD parameters to map corresponding PF parameters. As a result 707x1024 matrix of PF parameter was prepared for training of NN as input data set. Here 1024 does not have any specific value, it is related with range of the radar and its value is only a resolution issue. To train the NN 10, 15, 20 and 25 range samples were chosen from this input data set. This is for observing the effect of different number of samples on results. As a result, dimension of targeted data set of NN became unchanged as 707x4, but the dimension of input data sample set varied as 707x10, 707x15, 707x20 and 707x25. Dimension of data sets and training logic of NN is shown in Fig. 5.
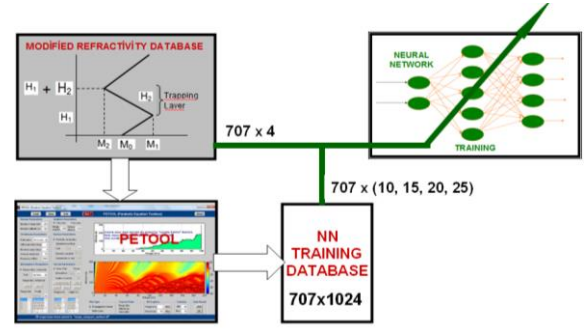

Fig. 5. Dimension of data sets and training logic of NN.

6. Inversion of PETOOL Software

Inversion capability of trained NN was tested by two ways:
1. SBD parameters which exist in training data set. This was done to see the inversion capacity of the NN.
2. SBD parameters which does not exist in training data set. This was done to see the generalizing capacity of the NN.

Range samples from input data set applied to NN for each time as 10, 15, 20 and 25. Each application also was repeated ten times to observe the stability of the results.

6.1. Inversion with Data Set Variables

A specific SBD condition is chosen from targeted data set to test the inversion capacity of the NN. Chosen variables and results obtained from NN inversion are given in Table 3 and Fig. 6. Ten trials were executed for each variable. Refractivity variables in Table 3 are obtained from arithmetic average of ten trials. Figure 6 shows results for 4 different variables; M_1, M_2, H_1 and H_2.

Generally, we can say that that the results for M_1, M_2, and H_2 are hopeful. They are around the target data with tolerable fluctuations. Nevertheless it will be useful to consider the results of H_1. The obtained results of H_1 are fluctuates mostly from 50 to 250. This may result from the data set of H_1, whose data

resolution is low when it compared to H_2. The most stable result from H_2 inversion is obtained for 10 samples inversion, which is the smallest sample number as shown Fig. 6b. And most unstable one is 25 samples inversion, which is the biggest number of sample. This was surprising, because we expected that increasing number of sample would improve the results.

Table 3. Reference duct variables, from Training Data Set and obtained variables.

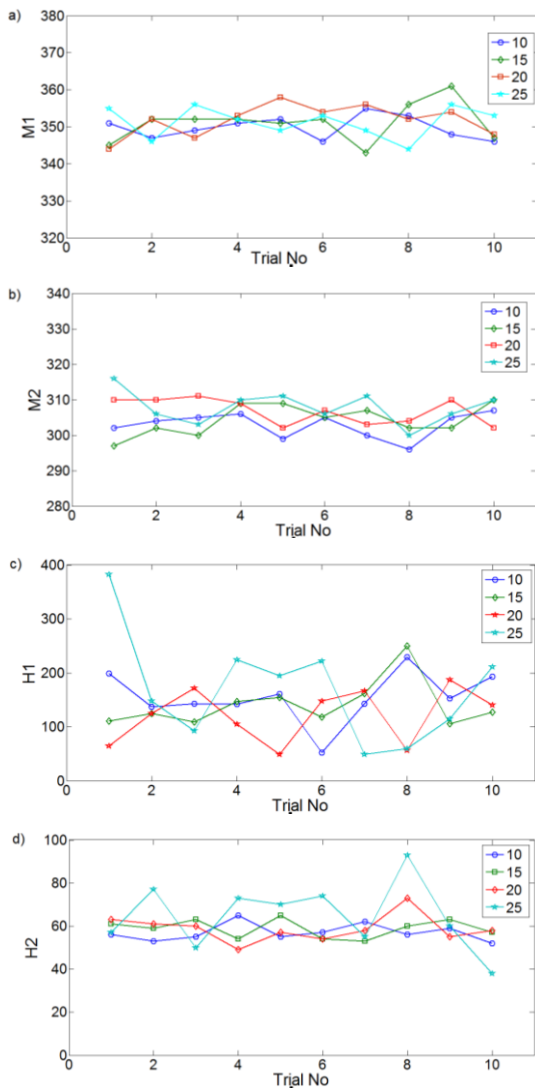|	M_1	M_2	H_1	H_2
Reference	350	310	170	60
Obtained (10 samples)	350	303	155	57
Obtained (15 samples)	351	304	141	59
Obtained (20 samples)	352	307	121	59
Obtained (25 samples)	351	308	170	65

Fig. 6. Inversion results of NN. (a) M_1; (b) M_2; (c) H_1; (d) H_2.
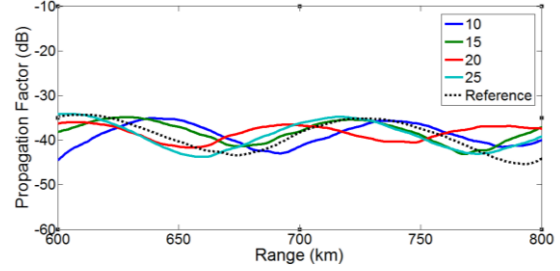

Fig. 7. Reference PF and PFs produced from the obtained refractivity variables by NN inversion method.

Fig. 7 shows the PFs produced from the obtained refractivity variables by NN inversion method and reference PF. The arithmetic average of each obtained duct parameter was taken to produce the obtained PFs. Reference PF is produced from the reference duct variables in Table 3. The shape of obtained PF curves is very similar to the reference PF. However, especially the result with 20 samples is at opposite phase to reference PF. There are also small phase differences between the other PFs and reference PF. These differences can be regulated easily by producing more output than 10 trials. For example 50 trials can be produced by inversion instead of 10 trials and the results can be purified from extreme ones, therefore it seems that it is possible to get more accurate results.

6.2. Inversion with Random Variables

A random duct condition is chosen which does not exists in data set. Reference variables that represents random duct condition and results obtained from neural network inversion are given in Table 4. Refractivity variables given in Table 4 are obtained from arithmetic average of ten trials.

Table 4. Reference duct variables, chosen randomly different from Training Data Set and obtained variables.

	M_1	M_2	H_1	H_2
Reference	365	295	400	45
Obtained (10 samples)	362	306	392	64
Obtained (15 samples)	366	302	389	71
Obtained (20 samples)	372	298	423	88
Obtained (25 samples)	373	297	413	78

These variable are though for NN inversion, because they are chosen near to edge of the data set. This is for forcing the capability of NN to understand its inversion limits. Fig. 8 shows the detailed results for 4 different variables; M_1, M_2, H_1 and H_2. Ten trials were executed for each variable. When we look at the results from random variables, M_1 and M_2 values are fluctuating more than the ones which were obtained from Data Set Variables. But H_1 is quite stable especially obtained from range samples of 10 and 15.

Fig. 9 shows reference PF, produced from reference duct variables in Table 4 and PFs produced from the obtained refractivity variables by NN inversion method. The arithmetic average of each duct parameter was taken to produce the PFs. The shape of PF curves that we obtained from NN inversion is very similar to the reference PF.
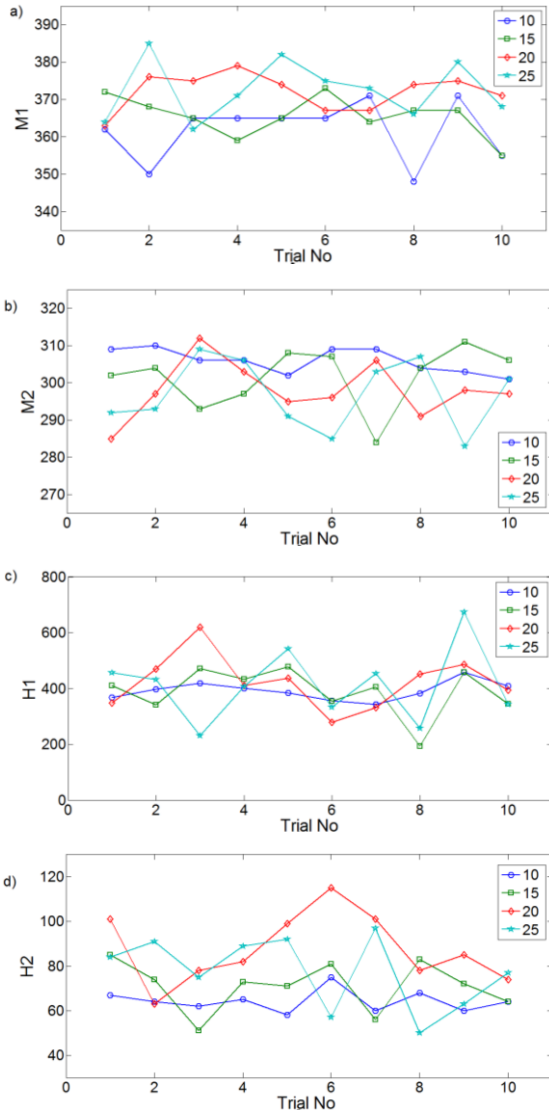
Fig. 8. Inversion results of NN. (a) M_1; (b) M_2; (c) H_1; (d) H_2.
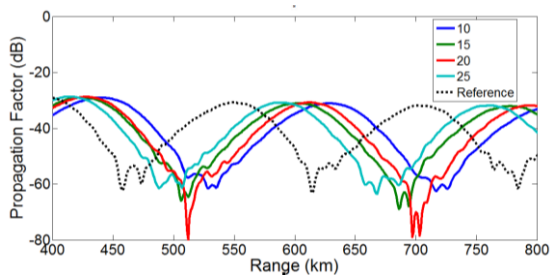

Fig. 9. Reference PF and PFs produced from the obtained refractivity variables by NN inversion method.

There is a phase difference between the lines of reference PF and obtained PFs as seen in Fig. 9. This may result from a shift of only one of the four variables (M_1, M_2, H_1 and H_2). Because, each variable may be effective on the shape of PF. After analysing and detecting the actual reason of these shifts, a correction algorithm can be employed to reach the exact refractivity variables which will be final results of the whole Refractivity Estimation Process.

7. Conclusions

In conclusion, NN is a strong candidate method for inversion in RFC techniques. NN can be applied for derivations refractivity variables from real radar clutter data. It is possible to see the learning and generalization capability of NN in the inversion process as obtained results, especially in Table 3 and 4. NN can be used to produce near real time refractivity parameter from radar data, providing that a powerful clutter model is used in the calculations. This advantage of NN can be used in design of radar that will be able to track the atmospheric refractivity changes by using RFC technique. Studies that we implemented here showed that some improvements in NN training data set may result in better estimation of refractivity parameters. Especially H_1 variables can be arranged so as to represent base height of surface based duct more accurately. Another improvement must be performed in assessment of exact refractivity parameters that were obtained from inversion. Totally, it can be said that NN is a good alternative as an inversion technique for RFC method with some improvements.

8. References

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