Multi-agent Formation Control with Single Integrator Agent Dynamics

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Abstract

In this paper, we used algebraic connectivity in a network to speed up formation in multi-agent autonomous systems. Formation topology of a multi-agent system with single integrator agent dynamics represented as a graph problem. Algebraic connectivity of the graph is expressed as an optimization problem where the objective is to maximizing it through the graph Laplacian. The problem is solved for some formation examples. Simulation is verified that the multi-agent system with maximized algebraic connectivity network structure (optimized Laplacian structure) has more convergence speed in reaching to the given formations.

1. Introduction

Multi-agent system's coordination and control through an inter-agent communication network is a challenging issue. The communication based systems have raised a number of important applications such as computer networked systems (Internet, LANs and WANs), sensor networks, traffic control systems, smart grid based systems, multi-robot systems, etc. In networked systems, the main problem is the lack of global interactions due to the hardware and/or software limitations or highly costs [1]. Consequently, it is desired to develop control protocols for network systems without the need of global properties

Graph theory is a common mathematical way for the analysis of dynamic networked systems in which the, inter-agent interactions inside a networked system is represented with the associated graph geometry. The network nodes and inter-node communication links are identified with graph nodes and edges. This representation provides the possibility of analyzing local interaction topology with related sub-graphs or special graph matrices.

Most of the networked systems encounter with stability, robustness and convergence problems which arise various topologies in different environments. In formation control problem, a multi-agent system with the given number of physical entities or simply agents that have communication with each other come together to achieve a desired formation (shape) task. The convergence properties of a multi-agent system with a given network structure can be analyzed through graph topology and Laplacian matrix. The eigenvalues and eigenvectors of this matrix are prominent parameters to evaluate the connectivity properties in a networked system. The second smallest eigenvalue is known as *Fiedler* eigenvalue [2] and it used as algebraic connectivity measure in a network. In a network the magnitude of algebraic connectivity used to analyzing the convergence speed, robustness and synchronization of the network. Since the connectivity, robustness and convergence are directly related in a network with its connectivity [3, 4], a general approach to evaluating a networked system properties is the optimized design of the network topology (communication structure).

The papers in the literature mostly focused on the Laplacian matrix and Fiedler eigenvalue as the network convergence speed parameter [3, 4, 5, 6]. The main approach was to maximize the Fiedler eigenvalue [3]. In [7, 8], edge weight values of graphs are manipulated to obtain Laplacian matrix with maximum Fiedler eigenvalue. A second approach is the manipulation of the edge connections, such that adding or removing some edge, desired graph topology is formed [3, 9]. In this paper, the first approach is used to speed up the convergence of agent states to steady-state for a formation.

Formation is a nature inspired behavior for autonomous multi-agent systems such as multi-robot systems. Some animal groups prefer collective motions due to different purposes such as finding food, encountering with predators, or migration. The group utilizes a formation behavior in the form of geometrical shapes, such as V-shape group motion in bird flocks .Similarly to the collective motions of animals, in the multi-agent system, formation can be formed and kept through tuning the control inputs subjected to limitations on the available information to each agent. Formation control problem is a main issue in a wide array of robotics applications such as indoor-outdoor exploration [10], manipulation [11], military multi-robot systems [12], etc. Also, inspiring from quadrupeds, birds, and fish, establish of formation in Unmanned Ground Vehicles [12, 13], Unmanned Aerial Vehicles (UAVs) [14], and Unmanned Underwater Vehicles [15] are studied.

In order to analyze and design control strategies for such interactive systems, individual agents dynamics and inter-agents information exchange network topology are important parameters. Agents with linear dynamics are mostly preferred for multi-agent control studies. Single integrator dynamics models are appropriate if the velocities of the mobile agents can be directly controlled [16]. This study also deals with the formation control of a multi-agent team with single integrator agent dynamics and distributed (neighborhood) information exchanging problem.

This paper is organized as follows: the problem formulation including agent dynamics, basic graph theory and the proposed optimization problem to maximize Fiedler eigenvalue are stated in section 2. In section 3, the problem solution for some examples is presented. Results and conclusions are stated in Section 4.

2. Formation Statement

2.1. Agent Dynamics

Consider a group of N mobile agents, where all agents are in accordance with single integrator dynamics. The group motion is assumed in two dimensions. For agent i $(i = 1 \ldots N)$ state and control vectors are $z_i = [q_{i1}, q_{i2}]^T$ and $u_i = [u_{i1}, u_{i2}]^T$, respectively $(z_i, u_i \in R^2)$. Single intergator or first order dyanmics for agent i

$$\dot{z}_i = u_i \,. \tag{1}$$

where u_i is velocity input. The whole group state and control vectors can be represented as $z = [z_1, \ldots, z_N]^T$ and $u = [u_1, \ldots, u_N]^T$, respectively $(z, u \in R^{2N})$. Then, the group dynamics is

$$\dot{z} = u \,. \tag{2}$$

Let $z_i^d = [q_{i1}^d, q_{i2}^d]^T$ be the desired state vector for agent i and then desired group state vector can be described as $z^d = [z_1^d, \ldots, z_N^d]^T$, $(z^d \in R^{2N})$. The desired state z_i^d also described with first order dynamics (1).

2.2. Graph Representations and Laplacian Matrix

Graphs are used to represent the networked systems. In the formation control of a multi-agent system, a vertex (node) and edge set represents the group agents and the information network connections among them. Let vertex and edge set is $V = \{v_1, \ldots, v_N\}$ and $E = \{(v_i, v_j) \in V^2\}$, respectively, then the graph formation is represented as $G = (V, E)$. If information among agents (or graph nodes) is transmitted through unidirectional links, then the corresponding edge representation should be directed and the graph $G = (V, E)$ is directed. For the bidirectional transmissions, the graph $G = (V, E)$ is undirected. The graph $G = (V, E)$ is also simple, i.e. has no loops and multiple edges. Associating a weight value $w_{ij} > 0$ to every edge $(v_i, v_j) \in E$, the graph $G = (V, E)$ is an edge weighted graph. A necessary condition for formation control of a group is the information transmitting connectivity among the members. In graph representations, this is shown with a connected graph. Fig. 1 shows an undirected graph representation of a multi-agent autonomous system with five agents.

For an undirected graph $G = (V, E)$ the *Laplacian* matrix is defined

$$L = D - A. \tag{3}$$

where $A = [a_{ij}]$ and $D = [\delta_{ij}]$, $(A, D \in \mathrm{R}^{\mathrm{N}})$ such that $a_{ij} = w_{ij}$ if and only if $(i, j) \in E$ and $\delta_{ii} = \sum_j a_{ij}$.

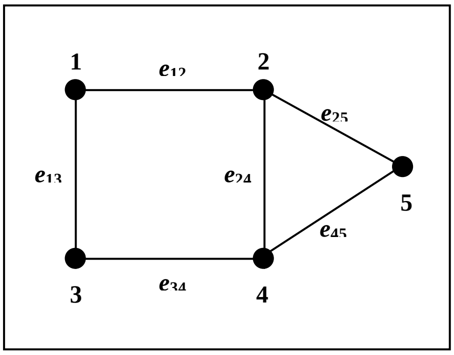

Fig. 1. An undirected graph representation for five agents.

In the Laplacian matrix L, only non-zero entries are in the intersections of rows and columns (i, j). Also, sum of all entries at any rows and columns are zero. The Laplacian L is symmetric, positive semi-definite. It has rank of $(N-1)$ and all of its eigenvalues are real that can be ordered as $\lambda_1 \le \lambda_2 \le \ldots \le \lambda_N$.

In the formation control, agents should keep a given distance with its neighbors that have connection. In two dimension coordinate, the desired distance between two node j and i is given as $d_{ij}^d = |z_j^d - z_i^d|$ where the team objective is to reach

$$|z_j - z_i| = \left((q_{j1} - q_{i1})^2 + (q_{j2} - q_{i2})^2 \right)^{\frac{1}{2}} \to d_{ij}^d \,. \tag{4}$$

as time is infinite. The set of neighbors of node i

$$N_i = \{ j \in V : (i, j) \in E \}. \tag{5}$$

Therefore, the local formation (neighborhood) control protocol for agent i is

$$u_i = \sum_{j \in N_i} w_{ij} \left(z_j - z_i - d_{ij}^d \right). \tag{6}$$

Local protocol (6) is distributed in which the state information of the neighbors N_i of node i in the information network topology is used only. The protocol (6) drives all agents' states to reach to the prescribed formation. The next result states that the local formation protocol depends on the graph Laplacian matrix L and the formation is obtained with (6) as control input [17].

Theorem 1. Consider a group of N mobile agents with single integrator dynamics (1). The formation control input is dependent to the Laplacian matrix L

$$u = -L\left(z - z^d\right). \tag{7}$$

Proof. $d_{ij}^d = |z_j^d - z_i^d|$ is known and we want to show that protocol (6) solves the formation control problem.

$$
\begin{aligned}
\dot{z}_i &= \sum_{j\in N_i} w_{ij} \left| z_j - z_i - d_{ij}^d \right| = \\
& - z_i \sum_{j\in N_i} w_{ij} + \sum_{j\in N_i} w_{ij} z_j - \sum_{j\in N_i} w_{ij} \left(z_j^d - z_i^d \right) = \\
& - \left(z_i - z_i^d \right) \sum_{j\in N_i} w_{ij} + \sum_{j\in N_i} w_{ij} \left(z_j - z_j^d \right) = \\
& - \delta_{ii} \left(z_i - z_i^d \right) + \\
& \left[w_{i1}, ..., w_{iN} \right] \left[\left(z_1 - z_1^d \right), ..., \left(z_N - z_N^d \right) \right]^T
\end{aligned} \quad . \tag{8}
$$

Global dynamics are given as;

$$
\begin{aligned}
\dot{z} &= -D\left(z - z^d\right) + A\left(z - z^d\right) = \\
& -\left(D - A\right)\left(z - z^d\right) = \\
& -L\left(z - z^d\right)
\end{aligned} \quad . \tag{9}
$$

Then, the global control input vector is given as;

$$
u = -L\left(z - z^d\right). \tag{10}
$$

Lemma 1. In a group of N mobile agents with single integrator dynamics (1), the agents reach to the formation if and only if the local protocol (6) is applied as control input.

Proof. The Laplacian matrix L has at least one eigenvalue at $\lambda_1 = 0$. Moreover, when the system is at its steady state $(u = 0)$ the smallest eigenvalue of L is 0 with eigenvector $\vec{1} = [1,...,1]^T$

$$
-L\vec{1}c = 0 \, . \tag{11}
$$

where c is any constant. From (11), $\vec{1}c$ is a right eigenvector for $\lambda_1 = 0$ and, the steady-state is in the null-space of L . Since L has rank of $(N-1)$, then $\vec{1}c$ is the only vector in the null-space of L and

$$
z - z^d = c\vec{1} \, . \tag{12}
$$

is satisfied for some c . Relation $z_i - z_i^d = c$ implies the formation (with fixed inter-agent distance assumption such as c) that is reached.

2.3. Optimization Problem

The Laplacian matrix has key role in the analysis of multi-agent systems behaviour where the interagent's communication netwrok is available in graph reperesenation. Various graph topologies related to the formation shapes and information network structure can be analyzed by computing corresponding Laplacian eigenvalues. In other words, the eingenvalues depend on the graph topology. The Fiedler Eigenvalue of the Laplacian (λ_2) is directly related with the speed of interactions in the multi-agent system. The graphs with largest Fiedler eigenvalue are faster to converge for the connected graphs, anytime $\lambda_2 \geq 0$.

Lemma 2. Consider a group formation of N mobile agents where the group communication network is described with graph representation. For Laplacian matrix corresponding to the graph representation, the Fiedler Eigenvalue satisfies

$$
\lambda_2 \leq \left(\frac{N}{N-1} \right) \min{}_i \left| N_i \right| . \tag{13}
$$

Proof. See [18].

From lemma 2, it is clear that the formation related with the *complete* graph topology has the biggest λ_2 value and (13) is reduced to $\lambda_2 \leq N$. Also, in the formation because of its connectivity, the minimum possible value for $|N_i|$ is one so that Eq. (13) become $\lambda_2 \leq \left(N / N-1 \right)$.

Given a labeled weight set $W = \{w_1, ..., w_{|E|}\}$ with weight value $w_i > 0, \left(i = 1...|E|\right)$ is assigned to an edge in the labeled edge set $\Xi = \{e_1, ..., e_{|E|}\}$. Each edge label e_i means an edge $(i, j) \in E$. The size of W is same as the size of E . Here, the main aim is to order the weight set W to maximize the second smallest eigenvalue λ_2 in the Laplacian.

For this reason, the following optimization problem is proposed in this paper

$$
\begin{aligned}
& \max \lambda_2(L) \\
& \text{s.t.} \;\; w_{ij} \in W
\end{aligned} \quad . \tag{14}
$$

where w_{ij} is the weight of edge conneting nodes i and j . Since $\lambda_2(L)$ is used in computing $\lambda_2(G)$, then they can be used in analyzing λ_2 .

For a graph G with $|E|$ edge number and N nodes, the Laplacian matrix L can also be factorized as

$$
L = \Delta^T \Delta = \sum_{i=1}^{|E|} w_i \Delta_i \Delta_i^T \, . \tag{15}
$$

where w_i is the weight of edge i and Δ_i is a column vector of the incidence matrix $\Delta = \left[\Delta_1, ..., \Delta_{|E|}\right]$. After the derivation of (15), the optimization problem (16) is obtained:

$$
\begin{aligned}
& \max \lambda_2(L) \\
& \text{s.t.} \;\; L = \sum_{i=1}^{|E|} w_i \Delta_i \Delta_i^T \\
& \qquad w_{ij} \in W
\end{aligned} \quad . \tag{16}
$$

The equation (16) is a constrained optimization problem that can be solved by various techniques such as Semi-definite programming, Quadratic programming, greedy heuristics, etc [3].

3. Simulation Results

The simulations are carried out for a formation with five robots $(N = 5)$. The graph representation contains the node and the edge set labeled as {1, 2, 3, 4, 5} and {e_1:12, e_2:23, e_3:13, e_4:24, e_5:34, e_6:45}, respectively. We assume two different weight sets as W_1={1,2,3,4,5,6} and W_2={2,3,3,3,1,1}. Then, two different formation problems with different edge weight set should be considered. In both problems, the main aim is to find that which of the weight permutations gives maximized Laplacian λ_2 due to the optimization problem (16).

For the problems subjected to weight set W_1 and W_2, the solutions are obtained with the algorithm given in Fig. 2. The maximum Fielder values related with the weight set W_1 and W_2 are $\lambda_2^* = 3.8273$ and $\lambda_2^{**} = 2.2126$ with corresponding weight orders $W_1^* = \{3,1,2,5,4,6\}$ and $W_2^{**} = \{1,1,3,2,3,3\}$, respectively.

Fielder eigenvalues for all permutations of weight set W_1 and W_2 are presented in Fig.3 and Fig.4, respectively.

The maximum Fiedler values $\lambda_2^*, \lambda_2^{**}$ for both examples are used to form the optimized Laplacian matrices L^*, L^{**}. These matrices are applied to the formation control input protocol (7). The initial formation and the desired positions in two-dimensions are given as z= [1 1; 12 5; 15 10; 3 10; 10 1]^T and z^d= [2 2; 2 4; 4 4; 4 2; 6 2]^T vectors, respectively.

Here, Fiedler eigenvalue λ_2 is used to interpret the time to reach to the steady-state in the formation which means that the desired formation is reached faster with the largest λ_2. The agents' control input (i.e. control effort) to transfer the system to the steady-state is given in Fig. 5 for the graph topology with W_1^*. The total simulation time step is set to 200. Fig. 6 shows the agents' control input for $\lambda_2 = 2.4991$ and its corresponding edge weight order $W_1 = \{4,5,2,3,1,6\}$. Also, in Fig. 7 and Fig. 8 the agents' control input for the graph topology weighted with W_2^* and $\lambda_2 = 1.5725$ with corresponding $W_2 = \{1,3,3,3,1,2\}$ are given, respectively. All agents' control inputs to transfer the system from initial to desired positions are given in Table 1 at the end of the total time step for graph topologies with edge weights W_1^*, W_1, W_2^{**}, W_2.

Concluding from Fig. 5-8 and Table 1 it is seen that at the end of total time step, the control effort is minimized in the case of larger λ_2 which means that the system will reach to the formation in a shorter time.

```
READ Weight Set
FOR each permutation of the Weight Set
    COMPUTE Laplacian L
    COMPUTE second smallest eigenvalue of L
END FOR
DISPLAY MAXIMUM second smallest eigenvalue
among all permutation Laplacian L
DISPLAY corresponding Weight permutation
```

Fig. 2. The pseudocode used to solve the optimization problem.

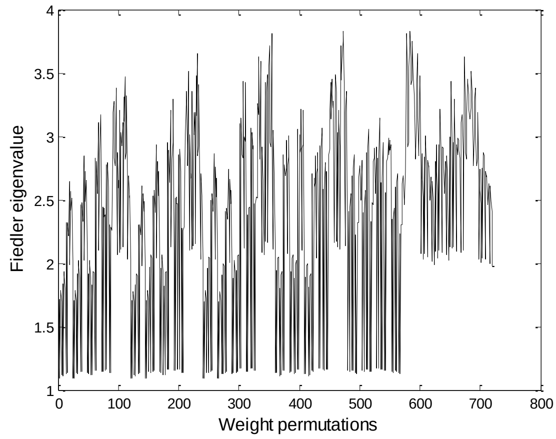

Fig. 3. The Fiedler eigenvalue distribution for all permutations of weight set W_1.

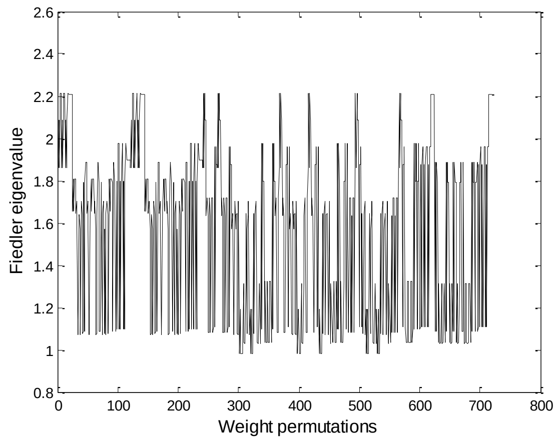

Fig. 4. The Fiedler eigenvalue distribution for all weight set W_2 permutations with the repetitions.

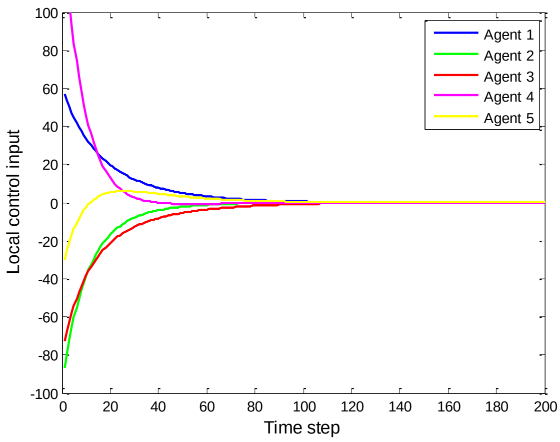

Fig. 5. Formation plot for graph topology with optimized edge weighted structure (in case of W_1).

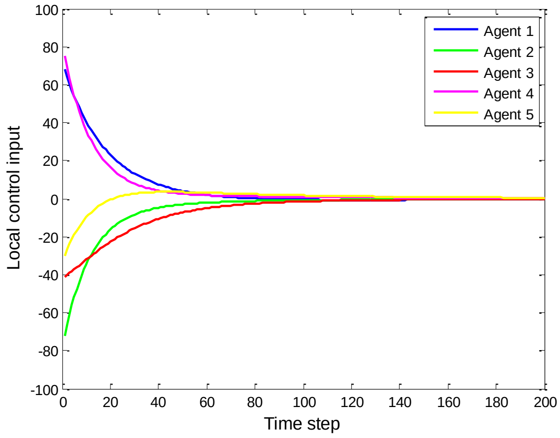

Fig. 6. Formation plot for graph topology with edge weighted structure W_1={4,5,2,3,1,6} corresponding to λ_2=2.4991 (in case of W_1).

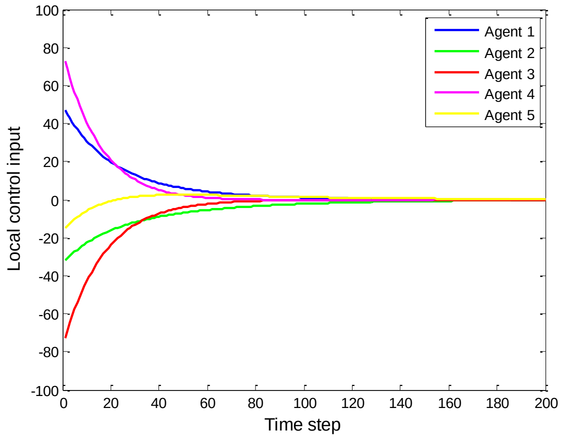

Fig. 7. Formation plot for graph topology with optimized edge weighted structure (in case of W_2).

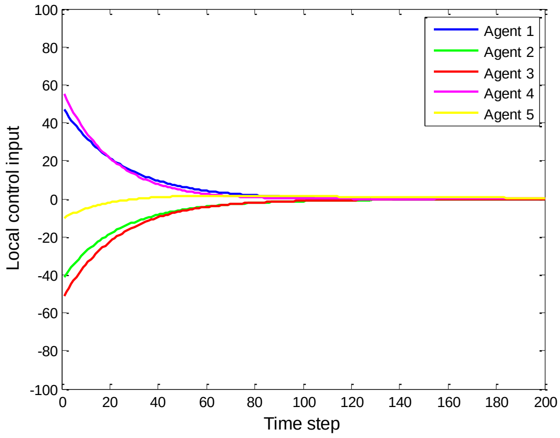

Fig. 8. Formation plot for graph topology with edge weighted structure W_2={1,3,3,3,1,2} corresponding to λ_2=1.5725 (in case of W_2).

Table 1. Agents' control inputs to transfer the system from initial to desired positions are given at the end of the total time step for graph topologies with edge weights W_1^*, W_1, W_2^{**}, W_2.

	Control input			
	W_1^*	W_1	W_2^{**}	W_2
Agent 1	0.0120	-0.3238	-0.0270	-0.1887
Agent 2	8.0154×10^{-4}	-0.1767	-0.2681	-0.1587
Agent 3	-0.0149	-0.2368	-0.0400	-0.1950
Agent 4	-0.0031	0.2705	0.0295	0.0397
Agent 5	0.0051	0.4668	0.3057	0.5026

4. Conclusions

This paper proposes a strategy to speed up formation in multi-agent autonomous systems based on algebraic connectivity in networks. The system is modelled with single integrator dynamical agents where the communication network is represented by graph topology. The process of obtaining optimal graph structure related to a fast formation topology is described with an optimization problem. Laplacian eigenvalue is used as the core of the problem. Finally, a simulation study is designed to show that the elements or structure of Laplacian matrix can be used as a basis for discussion or interpretation of multi-agent system's formation.

5. References

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