# LQG/LTR Position Control of an BLDC Motor with Experimental Validation

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## ABSTRACT

**In this paper, position control of a BLDC motor is studied. This position control is LQG/LTR control algorithm. In addition, a system identification approach is used to obtain the nominal plant of BLDC Motor. As a consequence, proposed controller is employed for an experiment. It is done by a real-time target machine.**

**Keywords: LQG/LTR, position control, BLDC motor, control, system identification, real-time operations.**

## INTRODUCTION

This paper studies the position control of guided system's flap that actuated by BLDC motor under unknown disturbances. The reasons behind using BLDC motor for flap control are summarized as follows. BLDC motors have better speed-torque characteristic, better dynamic performance, better efficiency, longer shelf-life than DC motors [1]. According to these advantages, permanent magnet brushless DC motors have been used increasingly in aerospace, military and industrial applications for years [2].

On the other hand, control of the system's behavior is not easy to handle for aerospace and military applications because of environmental conditions [3]. In addition, the dynamic structure of BLDC motor is highly complicated and has non-linear properties [4]. Moreover, the performance specifications of mentioned applications are relatively high and the modeling errors and plant uncertainties can affect negatively the performance of the flaps and related to system's overall performance.

To achieve precision operation and meet high performance specifications, it is necessary to develop a controller that has good robustness properties and overcome parameter variations, plant uncertainties and load disturbances. These controller requirements can be made by several robust controllers [5-6]. Among them LQG/LTR methodology is well-developed, well-known and easy-to-use [7]. The main aim of LQG/LTR methodology is the recovery of the desired robustness properties of Linear Quadratic Regulator in LQG design. Extended description can be found as in [8].

By using LQG/LTR methodology, the loop transfer function can be shaped so that the closed-loop system will yield;

i. Good command following

ii. Good disturbance rejection

iii. Good robustness

In this paper, LQG/LTR control system is employed to design the servo control loop of a permanent magnet brushless DC motor for meet the performance requirement of the overall systems and improve the robustness/stability performance in frequency domain.

This paper is organized as follows. Section II gives the general properties about dynamics of BLDC motor. The simplified mathematical model of the motor is introduced. By system/parameter identification, the approximate value of the parameter is found in this section. This nominal model is sufficient to design LQG/LTR compensator to achieve performance and stability specifications that are explained above. Section III presents the LQG/LTR compensator design. In this section, first, general mathematical explanations are given. Then, the desired loop transfer function is developed. This desired loop transfer function is focused especially in high frequency region since the simplified nominal model does not capture all relevant high frequency of the physical process. Finally, the complete control system design and analysis are done. Section IV illustrates the real experiment under acceptable conditions. The applicability of the LQG/LTR control theory in the design of position control for flap system is shown in this section. Moreover, experimental results are given and argued. Conclusions are given in Section V.

## DYNAMICS OF BLDC MOTOR

### *A. Modelling of PM BLDC Motor*

The flux distribution in PM BLDC motor is trapezoidal and therefore the d-q rotor reference frames model developed for PM synchronous motor is not applicable [9]. It is convenient to derive a Permanent Magnet BLDC motor model in phase variables for the nonsinusoidal flux distribution. The complete state-space representation of BLDC motor that includes nonlinear terms, as in (1)-(5), is given.

$$\dot{x} = Ax + Bu, \quad y = Cx + Du \tag{1}$$

$$x = [i_{as}\ i_{bs}\ i_{cs}\ w_m\ \theta_r] \tag{2}$$

$$A \begin{bmatrix} -R_s/L & 0 & 0 & \lambda_p f_{as}(\theta_r)/L & 0 \\ 0 & -R_s/L & 0 & \lambda_p f_{bs}(\theta_r)/L & 0 \\ 0 & 0 & -R_s/L & \lambda_p f_{cs}(\theta_r)/L & 0 \\ \lambda_p f_{as}(\theta_r)/J & \lambda_p f_{bs}(\theta_r)/J & \lambda_p f_{cs}(\theta_r)/J & -B/J & 0 \\ 0 & 0 & 0 & P/2 & 0 \end{bmatrix} \tag{3}$$

$$B = \begin{bmatrix} 1/L & 0 & 0 & 0 \\ 0 & 1/L & 0 & 0 \\ 0 & 0 & 1/L & 0 \\ 0 & 0 & 0 & -1/L \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{4}$$

$$u = [v_{as}\ v_{bs}\ v_{cs}\ T_l]^T \tag{5}$$

where $\theta_r$ is rotor position, $w_m$ is rotor speed, P is the number of poles, $f_{as}(\theta_r)$ is a magnitude limited function that shapes the instantaneous-induced emfs, $\lambda_p$ is peak mutual flux linkage, L is self inductance per phase, $R_s$ is stator resistance per phase, $v_{as}$ is input voltage for phases, $T_l$ is load torque, J is rotor inertia and B is damping coefficient. Above state-space realization is derived based on the assumptions that the induced currents in the rotor are neglected and iron, stray losses are neglected. The detailed derivations of above state-space representation are given as in [10].

The drive scheme of permanent magnet BLDC motor is simple and shown in Fig. 1. In this drive scheme, the encoder gives absolute rotor position. $T_e^*$ and $I_p^*$ are named as reference torque command and reference current magnitude command respectively. In addition, in a past few decades, various methods have been proposed so as to control each phase's currents [11-14]. In this paper, PID based current control system is used to regulate phases' currents.

## B. Simplified Mathematical Model of PMBLDC Motor

It is necessary to use a simplified model for control system design although simplified model does not cover the complete nonlinear system model that introduced above. This advantage is provided by the robustness of designed controller. The neglected dynamics of real system are compensated and taken as uncertainties if the designed controller's robustness properties are good enough.

The schematic of single phase PMBLDC motor is shown in Fig.2. This schematic contains position controller, which is the main purpose of this paper, current limiter that is caused by current saturation, current controller which regulates phases' currents, and converter model that is derived from electronic hardware. This scheme also contains simplified motor's mechanical and electrical models.

It is assumed that the motor operating in its constant-torque operation region and the flux producing current component is fixed. By these assumptions, simplified and equivalent model that very similar to a DC servo drive can be obtained and given in Fig.3. It should be noted that the equivalent and simplified model contains BLDC motor dynamics, inverter and current controller dynamics. In addition, the transfer function of this simplified motor model is given in (6).

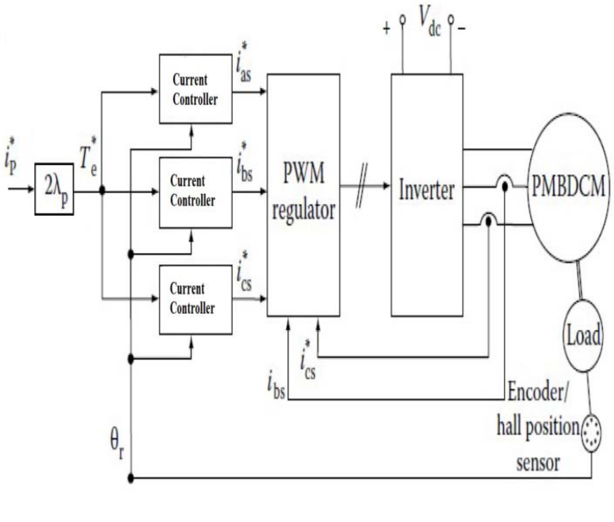


**Figure. 1.** Drive scheme of permanent magnet BLDC motor

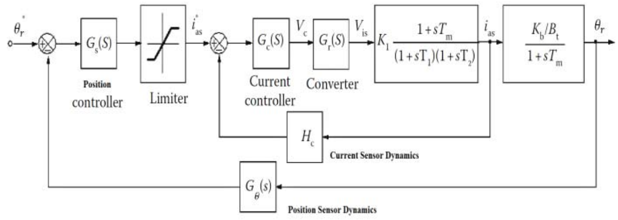


**Figure. 2.** Schematic of single phase PMBLDC motor

$$G_{plant}(s) = \frac{\theta_r(s)}{I_p^*} = K_{cc}\frac{k_t}{s^3LJ+s^2(RJ+BL)+s(BR+k_bk_t)} \quad (6)$$

where $K_{cc}$ is equivalent current controller, $k_b$ is equivalent back-emf constant and $k_t$ is torque constant.

## C. System Identification

The system identification methodology is a well-known process and widely used in literature. This process is useful when the system has a complicated dynamic [15]. In this paper, system identification process is used between the reference magnitude current $I_p^*$and system's actual position $\theta_r$. The applied input signal to $I_p^*$ is selected as swept sine signal shown in Fig.4. This signal contains all interested frequencies for BLDC motor operations. The definition of the applied input signal is given in (7).

$$u(t) = Asin(2\pi f_i(t)t + \emptyset) \quad 0 \le t \le T_0 \quad (7)$$

where $f_i(t)$ is the instantaneous signal frequency of input signal. $T_0$ is the signal's period and $\emptyset$ is the initial phase value. Definition of $f_i(t)$ is given in (8).

$$f_i(t) = f_{start}\beta^t, \quad \beta = (f_{end}/f_{start})^{1/T_0} \quad (8)$$

The parametric system model is obtained by MATLAB®/System Identification Toolbox. Bode diagram of defined system from $I_p^*$ to $\theta_r$ is shown in Fig.5. It is clear to say that the defined system has no unstable poles or zero.

# LQG/LTR CONTROL SYSTEM DESIGN

The dynamics of the plants is defined above. This dynamics shows that the plant is minimum phase and stable. These conditions are needed to achieve a good recovery of LQG controller [16]. Nominal system plant $G_{plant}(s)$ is augmented with PI elements for a good reference tracking. New plant's transfer function which is augmented by PI element is given in (9).

$$G(s) = G_a(s)G_{plant}(s) \quad (9)$$

where $G(s)$ is augmented plant and $G_a(s)$ is augmentation dynamics. The LQG/LTR compensator is designed for augmented plant $G(s)$.

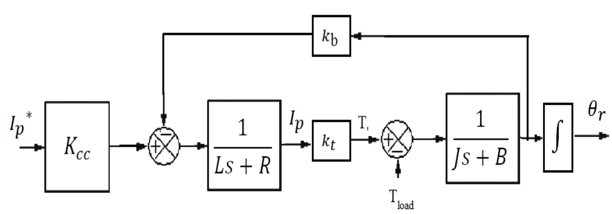


**Figure. 3.** Simplified and equivalent DC motor model

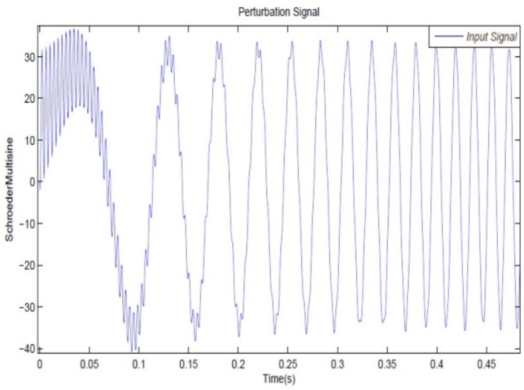


**Figure. 4.** Swept sine signal for system identification

The proposed system that includes integral augmentation and LQG/LTR control system is shown in Fig.6. In this system $G_{LQG/LTR}(s)$ is the proposed control system.

In LQG/LTR design methodology, the poor properties of LQG control system is recovered at plant input or plant output [7-8]. In this works, input recovery methodology is preferred and the complete control system design procedure is given step-by-step below.

1. System identification procedure defined above.
2. Determine the Target Feedback Loop (TFL).
3. LQG/LTR design

## A. *Determine the TFL*

The target feedback loop must have good robustness and performance properties. That requirement is done by LQR approach. This feedback loop is used to allow the motor to follow reference command with no steady-state error. The open loop transfer function of TFL is given in (10).

$$L_{input}(s) = K_{LQR}(sI - A)^{-1}B \qquad (10)$$

LQR guarantees some good properties that defined in (11).

$$|\hat{S}(jw)| \leq 1, \quad |\hat{T}(jw)| \leq 1 \qquad (11)$$

where $\hat{S}(jw)$ is sensitivity function and $\hat{T}(jw)$ is complementary sensitivity function. The TFL overcome the plant uncertainties and unmodelled dynamics because of these two guaranteed equations.

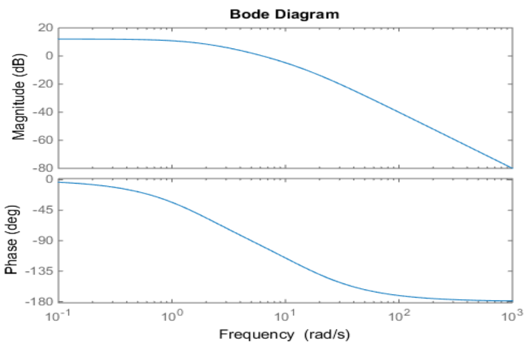


**Figure. 5.** Bode diagram for studied system

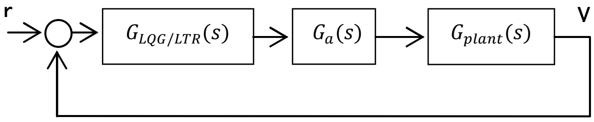


**Figure. 6.** Proposed control system with integral augmentation

In LQR design, the selected cost function is determined in (12).

$$J = \int_0^\infty (y^T Q^* y + u^T R^* u)dt \qquad (12)$$

where $Q^*$ and $R^*$ is positive semi-definite weightiness matrices. The main point of the selection of these weightiness matrices is the ratio of $\frac{Q^*}{R^*}$ [17]. In this work, $\frac{Q^*}{R^*}$ ratio is selected as 2e2. Determined TFL's open loop bode diagram, sensitivity function (S) and complementary sensitivity function (T) are given in Fig.7. Solution of minimizing problem of cost function J is given in [18].

## B. *LQG/LTR Design*

The schematic of LQG/LTR control system is given by Fig.8. Step-by-step design procedure is given below.

i. In this section, plant, that aimed to control by LQG/LTR approach, is augmented plant $G(s)$. The numeric data of this plant which in state-space form is given in Table 1.

ii. The design specification must be defined before the design of control system. The rules of these specifications are obtained from robustness-performance curve that is given in Fig.9. The cross-over frequency, sensitivity function and complementary sensitivity function obtained by using this curve.

iii. This step includes the LQR controller design is defined above. The solution of LQR problem is required the solution of Ricatti Equation (13).

$$A^T P + PA - \frac{\rho}{\beta} PBB^T P + C^T C = 0$$

$$R := D^T D + \frac{1}{\rho}, \; Q := C^T C \qquad (13)$$

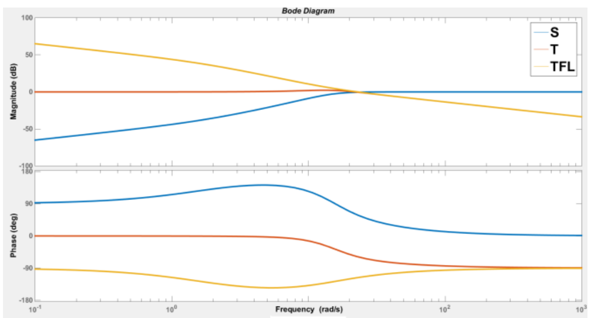


**Figure. 7.** Target feedback loop, sensitivity function and complementary sensitivity function diagrams

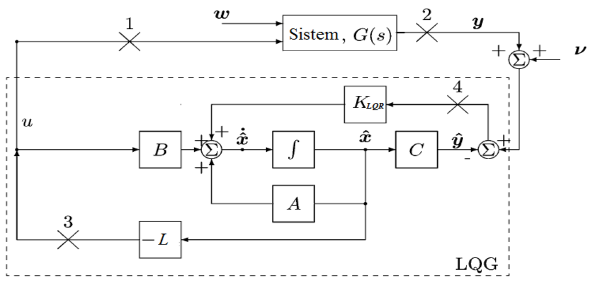


**Figure. 8.** Schematic of LQG/LTR control system

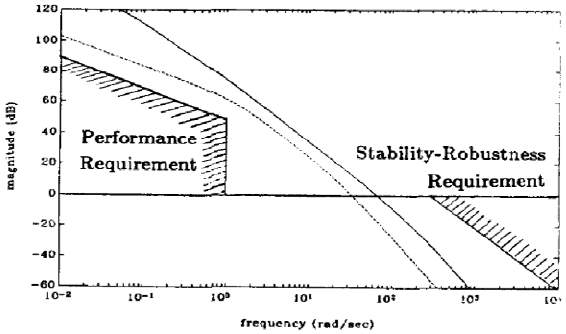


**Figure. 9.** Robustness-Performance curve

The selection of matrices C is arbitrary. The free parameter β is robustness parameter and in many application the β is selected as 2. The free parameter ρ is bandwidth parameter and for higher bandwidth, ρ must be selected higher.

iv. In this step, Kalman-Bucy filter design is done. By this step, $G_{LQG/LTR}(s)G(s)$ approaches $K_{LQR}(sI-A)^{-1}B$. To obtain loop transfer recovery another Ricatti Equation must be solve (14).

$$A^T\Sigma + P\Sigma - \Sigma C^T\mu^{-1}C\Sigma + BB^T = 0 \quad (14)$$

By this solution kalman gain (15) is obtained.

$$L = \sum C^T\mu^{-1} \quad (15)$$

The main free parameter in these equations is μ. For lower μ better recovery process is done.

v. Obtained the LTR controller is given in (16).

$$G_{LTR}(s) = K_{LQR}[sI - (A - BK_{LQR} - LC)]^{-1}L \quad (16)$$

However this form is not the final form. According to the plant augmentation the controller must be reform (17).

$$G_{LTR,final}(s) = G_{LTR}(s)G_a(s) \quad (17)$$

vi. Final step is applied the augmentation part. Anti-windup must be added all integral elements. The detailed derivation of these anti-windup algorithms can be found in [19].

In this section, the mathematical expression for the final operation is given in (18).

$$\lim_{\mu\to 0} K_{LQR}[sI - (A - BK_{LQR} - LC)]^{-1}LC(sI-A)^{-1}B \rightarrow K_{LQR}(sI-A)^{-1}B \quad (18)$$

The control system's parameters are given in Table 2. Moreover, the recovery of the controller for different values of μ is given in Fig.10. As a result of control system design, there is no unstable pole-zero cancellation that is one of the most important specification and all desired property of target feedback loop is achieved.

## EXPERIMENTAL SETUP & RESULTS

Experimental setup that is used for system identification and control system is given in Fig.11. Xpc target product of SpeedGoat Company is used for system identification and real-time control operations. 40 KHz PWM signal is produced by this product to achive current control. Desired position control loop updated up to 3 KHz. In this experiment, an EC-Motor of Maxon Motor Company is preferred. This motor has 171 Watt power rating and 28 V voltage supplied. All control algorithms is built in MATLAB/Simulink platform.

Step response of the designed position controller is given in Fig.12. and single phase's current shape given in Fig. 13. The system has neglicible overshoot and has no steady-state error. The overshoot that is occured in step response is caused by the system's unmodelled nonlinear dynamics. On the other hand, there is no unstability in the system althought the currents of each phases and mechanical system have uncertanity. The main objective of the designed controller is said to be succesful.

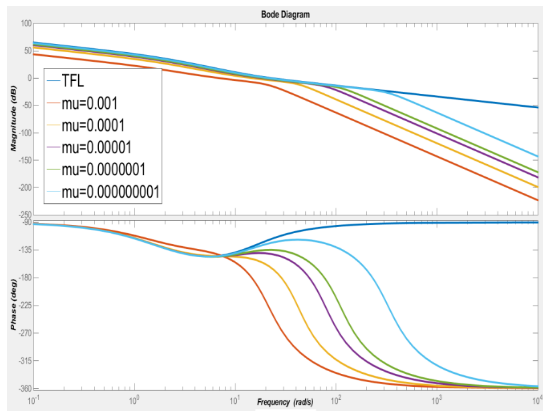


**Figure. 10.** Loop transfer recovery for different $\mu$ values

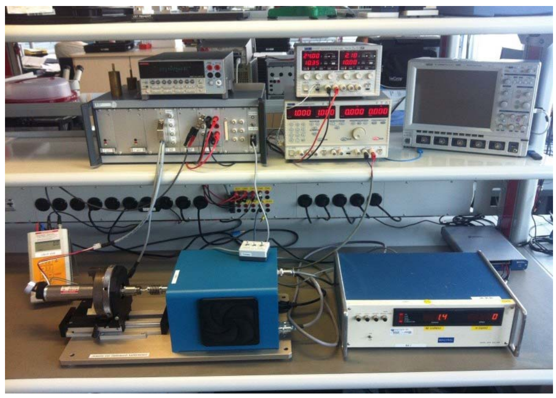

**Figure. 11.** Experimental setup

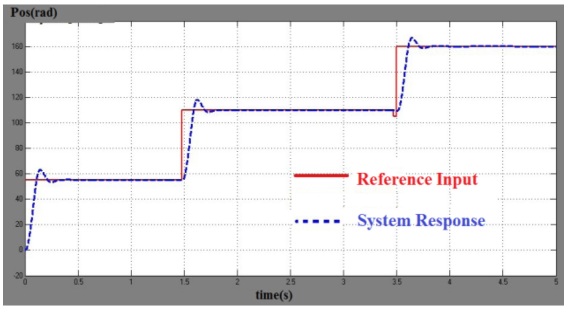


**Figure. 12.** Step responses of BLDC motor

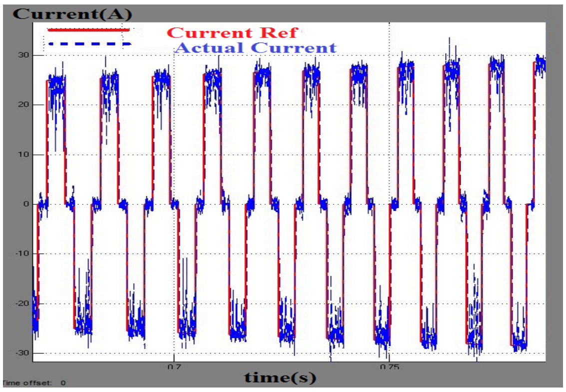


**Figure. 13.** Single phase current response

## CONCLUSION

In this work, a LQG/LTR position controller for BLDC motor and its experimental validation are presented. It is clear to say that the proposed control system can cancel unmodelled dynamics and system's uncertainities. Moreover, given control system is explained methodologically. The proposed control system meets many specification in robustness and performance. However, LQG/LTR linear control system is not sufficient for systems that require high performance specifications. In that case, other solutions like nonlinear controller design or cancellation of nonlinear effects must be spotted.

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