

# Solving Economic Emission Load Dispatch Problems Using Particle Swarm Optimization with Smart Inertia Factor

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## Abstract

This paper attempts to propose particle swarm optimization algorithm with smart inertia factor (PSO-SIF) to solve the problem of economic emission load dispatch (EELD) in thermal power plants. The aim of EELD solution is synchronous reduction of both fuel costs and emission level. EELD problem is a non-linear and non-convex problem which uses evolutionary algorithms as efficient to solve such problems due to its complicated characteristics. PSO-SIF algorithm is an efficient and robust algorithm to exploit universal optimal point of optimization problems. Tests implemented on 10-unite systems, considering valve-point loading effects, network losses with Multi-objective functions such as system fuel cost and emission level, to show PSO-SIF algorithm capabilities in solving EELD problems. Results obtained from present investigation indicated universal optimal point exploitation of the problems and superiority of the proposed algorithm compared to other new and efficient algorithms.

## 1. Introduction

The aim of economic load dispatch in thermal power plants is minimizing fuel costs in such power plants. Operating of Thermal power plants causes greenhouses gases production such as sodium oxide ( $SO_x$ ), nitrogen oxide ( $NO_x$ ) ( $x=1,2,3$ ) and etc., such emissions result in ecological concerns [1]. Using power plants with lower emission levels can happen synchronously with doing economic load dispatch. In this case, with combination of economic load dispatch problem and emission at objective function, a decision is made that can decreases both factors synchronously.

In some papers, emission is considered as a condition to solve EELD problem. Different studies have indicated that using a condition for emission causes difficulties in influencing EELD problem. Among these difficulties are difficulty of making relationship between fuel cost and emission [2]. Weighting algorithm give different weights to the functions, easily, based on their importance in the objective function. When solving the problem with weighting algorithm, the problem solution is with error due to dependency of objective function to function weight. Innovative algorithms don't have such problems, hence advanced algorithms such as genetic algorithm [3], evolutionary differential [4], particle swarm optimization [5], social biography [6], gravitational search algorithm [7], stock market algorithm [8] and etc., were developed to solve such problems.

An appropriate algorithm's particle swarm optimization is to solve optimization problems at large scales. Search method of this algorithm depends on the population and particles group. In 1995 PSO was proposed by Kenedy and Abr Heart based on analyzing behavior of fish and birds group [5]. Among the advantages of PSO algorithm are, simplicity, easy administration, its relative ability to continue running program and efficiency of its calculation system. However, PSO algorithm might trap due to inability in optimal search or inappropriate ability in making balance between universal optimal exploration and local optimal exploitation when confronting some problems and heavy conditions in local optimizations [9].

PSO-SIF [9] is a robust and strong improved version of particles swarm algorithm where, unlike PSO, weight inertia factor has not similar value for total population and its value is decreased in descending form with increase in iteration. Weight factor value is determined intelligently in PSO-SIF based on each individual's cost, standard deviation of cost of best group response, and in proposed algorithm each particle has its own inertia factor consequently individual speed for universal optimal point exploitation. Considering high abilities of the proposed algorithm in exploitation of universal optimal point problem, this paper tries to propose PSO-SIF algorithm as solution for EELD non-convex problem. In order to examine the efficiency of the proposed algorithm in solving EELD problem, present algorithm was implemented on a 10 unit non-convex system successfully, considering effects of pet cock (valve-point loading) and network losses. Results obtained from solution of above mentioned problems were compared with results of some robust and efficient algorithms. Results obtained from present investigation indicated higher efficiency and ability of proposed algorithm compared to other algorithms.

## 2. Formulation of Emission Economic Load Dispatch

Further information about EELD problem is available in [10-12], though generally the aim of EELD problem solution is reducing whole system's fuel costs along with its emission amount. Problem variables are produced powers of generators which can be defined as follows:

$$[P_G] = [P_{G1}, P_{G2}, \dots, P_{GN_g}]^T$$

Subjected to:  $h(P_{Gi}) = 0$  and  $g(P_{Gi}) \leq 0$

where  $N_g$  is the number of last generator,  $P_{Gi}$  is active generated power of  $i^{th}$  generator, and  $h(P_{Gi})$  is equality

condition and  $g(P_{Gi})$  is unequal condition of the studied problem. Power plants productivity is equal with total value of load and transmission line losses, in other words equality condition of studied problem is:

$$\sum_{i=1}^{Ng} (P_{Gi}) - P_{load} - P_{loss} = 0 \quad (1)$$

where  $P_{load}$  is load demanded power and  $P_{loss}$  is losses power in transmission line and its value for problem is obtained from following relation:

$$P_{loss} = \sum_{i=1}^{Ng} \sum_{j=1}^{Ng} P_{Gi} B_{ij} P_{Gj} + \sum_{i=1}^{Ng} B_{0i} P_{Gi} + B_{00} \quad (2)$$

In above relation,  $B$  is losses matrix.

Unequal condition of the studied problem is power plants productivity, being the range of a minimum and maximum, in other words:

$$P_{Gi \min} \leq P_{Gi} \leq P_{Gi \max} \quad (3)$$

Multi-objective function of EELD problem is:

$$\min F = [F_{FC}, F_E] \quad (4)$$

where  $F$  is multi-objective function of studied problem and the aim is to minimize it. Objective function of the problem involves fuel cost minimization of  $F_{FC}$  and Emission level minimization of  $F_E$  individually.

Objective function of power plant fuel cost is a quadratic function which is defined as follows:

$$F_{FC} = \min \sum_{i=1}^{Ng} (a_i + b_i P_{Gi} + c_i P_{Gi}^2) (\$/h) \quad (5)$$

where  $a_i$ ,  $b_i$  and  $c_i$  are consistent factors of thermal power plants' fuel cost. Fuel cost unit is dollar per hour ( $\$/h$ ). If there is any valve-point loading, then relation (5) is defined as follows:

$$F_{FC} = \sum_{i=1}^{Ng} (a_i + b_i P_{Gi} + c_i P_{Gi}^2 + |e_i \times \sin(f_i \times (P_{Gi,\min} - P_{Gi}))|) (\$/h) \quad (6)$$

where  $e_i$  and  $f_i$  are factors of power plants factors indicating valve point loading effects.

Objective function of power plants emission levels is a quadratic function defined as follows:

$$F_E = \min \sum_{i=1}^{Ng} (\alpha_i + \beta_i P_{Gi} + \gamma_i P_{Gi}^2) (t/h) \quad (7)$$

where  $\alpha_i$ ,  $\beta_i$  and  $\gamma_i$  factors are emission factors, and  $F_E$  shows emission level. Emission unit is ton per hour ( $t/h$ ) or kilogram per hour ( $k/h$ ). If there is valve point loading effects, relation (7) is defined as follows [10]:

$$F_E = \sum_{i=1}^{Ng} (\alpha_i + \beta_i P_{Gi} + \gamma_i P_{Gi}^2) + \xi_i \exp(\lambda_i P_i) (t/h) \quad (8)$$

where  $\xi_i$ ,  $\lambda_i$  and  $\beta_i$  show the effects of valve point loading.

### 3. Combined Economic Emission Load Dispatch

Emission and fuel costs functions are individual and independent functions. Fuel cost unite is ( $\$/h$ ) and emission

unit is ( $t/h$ ). Numerical value of each of these independent functions can be several times more than others. To combine this function and form a single objective function a decision should be made that while combining different functions unite, it would equiponderate their numerical value in objective function till the algorithm could consider the effects of all functions equally, when optimizing this objective function. Price penalty factor algorithm can convert independent functions to an objective function with the same unit. PPF algorithm [13] is chosen based on maximum fuel costs of power plants and maximum value of the emission is divided to it till all functions involve the same unit and weight in objective function. If objective function is formed of two separate functions of fuel cost and emission, the function of investigated problem will be as following relation, using combined PPF algorithm:

$$\min F = F_{FC} + PPF \times F_E (\$/h) \quad (9)$$

$$ppf[i] = \frac{\sum_{i=1}^{Ng} (a_i + b_i P_{Gi,\max} + c_i P_{Gi,\max}^2 + |e_i \times \sin(f_i \times (P_{i,\min} - P_{Gi,\max}))|)}{\sum_{i=1}^{Ng} (\alpha_i + \beta_i P_{Gi,\max} + \gamma_i P_{Gi,\max}^2) + \xi_i \exp(\lambda_i P_{Gi,\max})} (\$/h) \quad (10)$$

### 4. Review of Particles Swarm Optimization

PSO is a robust and efficient algorithm to exploit problems of universal optimal points. This algorithm is based on population which has been inspired from bird migration and fish groups [5]. In a search environment of next n, position and speed of particle  $i^{th}$  is shown with vectors  $X_i = (X_{i1}, X_{i2}, \dots, X_{in})$  and  $V_i = (V_{i1}, V_{i2}, \dots, V_{in})$ , respectively. Direction of these vectors show number of components and ( $P_{best} = X_{i1}^p, X_{i2}^p, \dots, X_{in}^p$ ) and ( $G_{best} = X_{i1}^G, X_{i2}^G, \dots, X_{in}^G$ ) indicate best position of particle  $i^{th}$  and best position of neighbors till now, respectively. Modified speed and position of each particle at the end of each iteration can be shown as follows:

$$V_i^{k+1} = \omega V_i^k + c_1 r_1 (P_{best}^k - X_i^k) + c_2 r_2 (G_{best}^k - X_i^k) \quad (11)$$

$$X_i^{k+1} = X_i^k + V_i^{k+1} \quad (12)$$

where  $V_i^k$  is the speed of particle  $i^{th}$  in  $k^{th}$  iteration,  $\omega$  is weight inertia factor,  $c_1$  and  $c_2$  are acceleration factors.  $r_1$  and  $r_2$  are random numbers from 0 to 1, and  $X_i^k$  shows particle  $i^{th}$  position at  $k^{th}$  iteration. Weight inertia factors are obtained for each iteration in PSO algorithm as follows:

$$\omega^k = \omega_{\max} - \frac{\omega_{\max} - \omega_{\min}}{iter_{\max}} \times k \quad (13)$$

where  $k$  is the iteration number and  $iter_{\max}$  is maximum number of iteration.

### 5. Particles Swarm Optimization with Intelligent Inertia Factor

Forming No balance between universal optimal exploration and optimal points exploitation is one of the most important factors in PSO algorithm trapping at local optimal points. With increase in iteration inertia, factor value is decreased in PSO which causes reduce in speed of particles, moreover if

algorithms can not exploit points which are close to optimal points at initial iterations, it will not, accordingly, be able to explore universal optimal point, successfully with decrease in particles' speed. This issue is the main reason of PSO algorithm convergence to unequal responses at each time of running program in some problems with heavier conditions [9]. PSO-SIF is an improved version of particles swarm optimization where using each population costs and its standard deviation value from best group response cost, required control is applied on inertia factor values. Considering that each particle cost is influential in the speed and position of next particle, so in our proposed algorithm each population will have individual inertia and speed in convergence to the best group answer. Following realties are used in PSO-SIF, to calculate cost of each population from best group cost [9]:

$$\delta = \frac{(\text{cost}_j - \text{cost}_{gbest})}{\text{cost}_{gbest}} \quad (14)$$

$$\lambda_j = \frac{\text{cost}_j}{\text{cost}_{gbest}} \quad (15)$$

where  $\text{cost}_j$  is  $j^{\text{th}}$  population cost, and  $\text{cost}_{gbest}$  is the best group response till now, and  $\lambda_j$  is change percent of  $j^{\text{th}}$  population cost from best group cost. In PSO-SIF,  $\lambda_j$  or cost rate can vary from 1 to  $1 + \delta_{\max}$ . It is considered that proposed inertia factor or  $\omega_g$  to have value of 0.3 to 0.9, based on cost rate. Hence, with calculation of both inequalities slope line, a linear relation can be formed among them, in other words:

$$0.3 \leq \omega_g \leq 0.9 \quad (16)$$

$$1 \leq \lambda_j \leq 1 + \delta_{\max} \quad (17)$$

$$\frac{\lambda_j - 1}{\omega_g - 0.3} = \frac{(1 + \delta_{\max}) - 1}{0.9 - 0.3} \quad (18)$$

$$\omega_g = \frac{0.6 \times (\lambda_j - 1)}{\delta_{\max}} + 0.3 \quad (19)$$

Relation (16) shows variation interval of proposed inertia factor and equation (17) shows variation interval of cost rate. Relation (18) shows linear slope equation of relations (16) and (17), and relation (19) is arranged form of relation (18). As it can be seen in relation (19), the only unknown of the equation is  $\delta_{\max}$  value or higher limit of  $\lambda_j$ . This shows that, in ratio of what  $\lambda_j$ , maximum inertia factor should be chosen.  $\delta_{\max}$  value in PSO-SIF algorithm is big value at initial iterations and this causes increase of search region and its value is decreased with increase of iteration. In other words,  $\delta_{\max}$  value is equal to following equation:

$$\delta_{\max} = \delta_1 - \left( \frac{\text{iter}}{\text{iter}_{\max}} \right) \times \delta_2 \quad (20)$$

where  $\delta_1$  and  $\delta_2$  are adjustable parameters of proposed algorithm and  $\text{iter}$  is program iteration number. An inertia factor is generated in ratio of each population and cost in PSO-SIF algorithm, so in proposed algorithm, each population has an inertia factor and consequently an individual speed for exploration of universal optimal point based on its cost. Fig. 1 shows generated inertia factor of fist and last population which have the best and the worst costs compared to PSO produced inertia factor.

The process of emission and economic load dispatch problem solution is based on PSO algorithm following these steps:

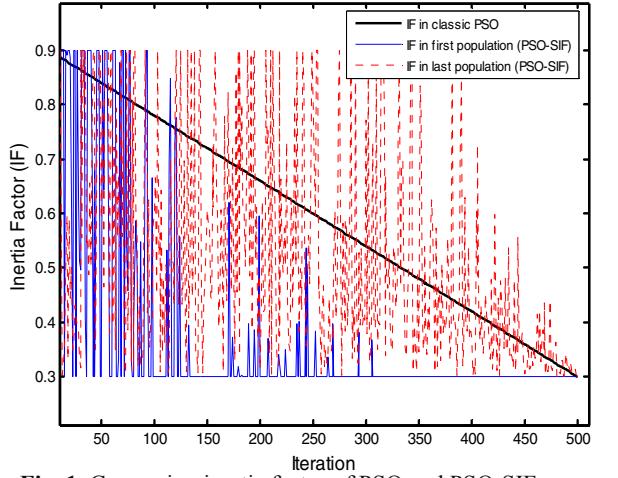
Step1- randomized production of initial population and initial speed of particles

Step2- calculation of cost of each population based on relation (9) and choosing the best group and individual response

Step3- Updating position and speed of particles based on relations (11-12)

Step4- improving new position of the particles to provide conditions of the problem

Step5- resuming program from step 2 till program ending criteria are provided



**Fig. 1.** Comparing inertia factor of PSO and PSO-SIF

## 6. Simulation Results

Different simulations were carried out on 10 unit systems produced with valve point loading, network losses with objective function including functions such as: non-smooth fuel costs and emission level. System total load was 2000MW and information of production units and network losses is available in (10).  $c_1$  and  $c_2$  values were selected 1.9 and 2.1 for proposed algorithm, respectively in all simulation.  $\delta_1$  and  $\delta_2$  values were 0.05 and 0.03, respectively. Number of initial population and number of program iterations were selected 100 and 1000, respectively. 50 independent tests were done to compare problem solution quality and convergence characteristics.

Simulations were carried out at 3 separately sections to minimize fuel cost, minimizing emission level and synchronous minimization of fuel cost and emission level. Simulation results were obtained under MATLAB software by PSO-SIF algorithm which presented on Table I. Results of EELD problem solution have been compared with strength pareto evolutionary algorithm (SPEA), non-dominating sorting genetic algorithm II (NSGA-II), multi objective differential evolution (MODE), pareto differential evolution (PDE) algorithms results by PSO-SIF in Table II. Fig. 2 shows the characteristics of synchronous convergence of fuel cost and emission in studied systems in ratio of fuel costs variations and emission level. As it can be seen fuel cost and emission level is at its highest level at initial

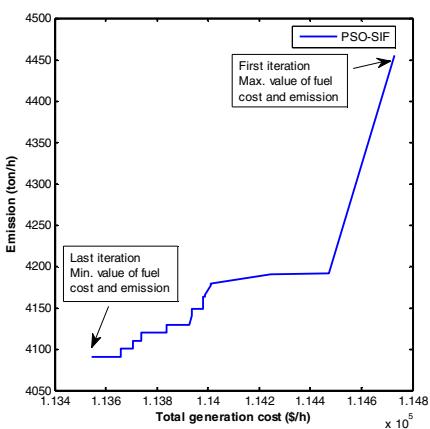
iteration and with increase in the number of iterations both values are decreased and at the last iteration of the program emission level and fuel cost are minimized synchronously.

**Table 1.** Simulation Results by PSO-SIF on 10 Generator System

Unit (MW)	Minimum fuel cost	Minimum emission	Best results for both
P1	55.0000	55.0000	55.0000
P2	80.0000	80.0000	80.0000
P3	106.6157	80.7795	85.5191
P4	100.7217	80.9836	84.1663
P5	81.3797	160.0000	143.0110
P6	83.3187	240.0000	162.9909
P7	30.0000	294.0945	298.5036
P8	34.0000	296.7213	314.2054
P9	470.0000	397.5342	428.5971
P10	470.0000	396.5199	431.8751
Total Power	2087.036	2081.6333	2083.8487
Power loss	87.036	81.6333	83.8487
Total Cost (\$/h)	111497.6560	116406.6914	113450.2966
Total Emission [t/h]	4571.2163	3932.2701	4111.3398

**Table 2.** The best simulation results by different methods on 10 generator system

Methods	PSO-SIF	MODE [10]	PDE [10]
Total Cost (\$/h)	113450.2966	113480	113510
Total Emission [t/h]	4111.3398	4124.90	4111.40
Methods	NASGA-II [11]	SPEA [12]	
Total Cost (\$/h)	113540		113520
Total Emission [t/h]	4130.20		4119.10



**Fig. 2.** The characteristics of synchronous convergence of fuel cost and emission

As it can be seen in Table I, minimum fuel cost obtained from PSO-SIF is 111497.6580 (\$/h) and emission value in this state is 4571.2163 (t/h). The least emission level obtained from

PSO-SIF is 3932.2701 (t/h). Results of synchronous minimization of emission and fuel cost can be seen in Table I and these results have been compared with other results in Table II. Least emission and fuel obtained in this way are 113450.2966 (\$/h) and 4111.3398 (t/h), respectively. When comparing these results with results of MODE algorithm in Table II, fuel cost and emission level obtained from PSO-SIF 29.7 (\$/h) and 13.56 (t/h) are minimum than MODE algorithm. As it can be seen fuel cost and emission level obtained from PSO-SIF algorithms 59.7 (\$/h) and 0.6 (t/h) is more minimum than PDE algorithm. Average time of running program for 1000 iterations is 11.45 seconds. Results of present simulation indicated superiority of proposed algorithm compared to other algorithms.

## 7. Conclusion

This paper proposes PSO-SIF algorithm for optimization of emission economic load dispatch problem. The proposed algorithm has higher capability in generating balance between universal optimal exploitation and local optimizations and consequently universal optimal point exploitation due to appropriate control and appropriate inertia factor initialization. Considering effects of valve-point loading and network losses the efficiency of proposed algorithm was studied with EELD problem solution in a non-convex system with 10 production units. Results obtained from studied problem optimization indicated higher ability of proposed algorithm in exploitation of universal optimal point compared to other efficient algorithms such as MODE, PDE, NASGA-II and SPEA.

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