Chaos Synchronization of the Fractional Order Time Delay System

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Abstract- In this paper, the chaos synchronization of two identical fractional order time delay system (One Master-Transmitter, other Slave-Receiver) has been achieved for different initial conditions. For achieving synchronization between two systems, a proportional controller has been designed by using active control method. Efficiency of the controller designed has been tested using modified Mikhailov stability criterion and its appropriate gain parameter has been selected. The obtained results have been verified by the time domain simulations of the system.

Keywords: Chaos synchronization, Fractional order model, Modified Mikhailov stability criterion, Fractional order transcendental characteristic equation.

1. Introduction

In 1990, after Pecora and Carroll showed that a chaotic system can be synchronized [1], because chaos is sensitive dependence on initial conditions and has unpredictable behaviors, chaos synchronization has been used in secure communication systems [2-4]. Therefore, synchronization of different chaotic systems has been achieved by various methods [5-7]. In addition, achievement of chaos synchronization of the fractional order chaotic systems with [8-9] and without delay [10-11] have been became a research topic.

Fractional order model of system given in [12] was obtained in [13-14] and it was shown that the model exhibits chaotic behavior. This fractional order system is defined the delayed fractional order differential equation with one dimension as follow:

$$\frac{d^q x}{dt^q} = \delta x_\tau - \varepsilon (x_\tau)^3 \tag{1}$$

where, q fractional order ($0<q<1$), δ and ε positive system parameters, τ is constant time delay ($x_\tau = x(t-\tau)$ and $\tau \in R^+$). System defined in Eq. (1) can exhibit chaotic behavior according to the system parameters.

Chaos synchronization of time delay system which integer order model of Eq. (1) and given in [12] has been performed in [6]. In this paper, a proportional controller has been proposed via active control method, which is given in [5], for chaos synchronization in two identical chaotic fractional order time delay system, one is Master (Transmitter) and other is Slave (Receiver), given in Eq.(1). In addition, with help of the modified Mikhailov stability criterion which is tested the stability of the fractional order linear system with delay [15], appropriate gain values of the controller has been determined. This method also has been used for bifurcation analysis of the fractional order nonlinear system with delay [16]. After this section, in Section 2, the required controller for the system synchronization has been specified via active control method and its modified Mikhailov stability function has been obtained. Section 3 has presented the computation of the controller gain at some system parameter. The results obtained in previous section have been verified by the time domain simulations in Section 4 and finally, in Section 5, the results have been discussed.

2. Synchronization of the System via Active Control

Fig. 1 presents two identical chaotic fractional order time delay system for synchronization via active control. Error dynamic between the Master and Slave system should be obtained.

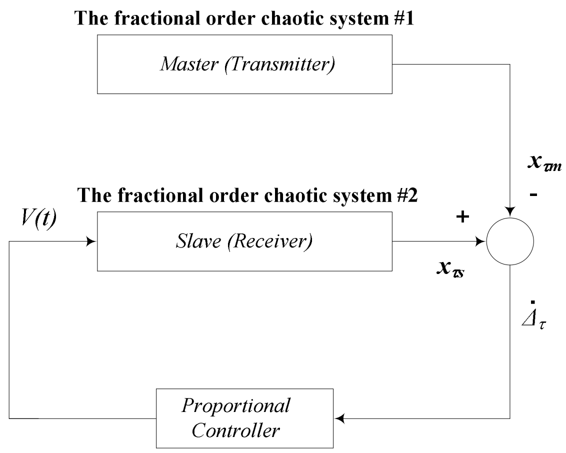


Fig. 1. Block diagram of the chaos synchronization.

Equations (2) and (3) define Master system and Slave system with control function respectively.

$$\frac{d^q x_m}{dt^q} = \delta x_{\tau m} - \varepsilon (x_{\tau m})^3 \tag{2}$$

$$\frac{d^q x_s}{dt^q} = \delta x_{\tau s} - \varepsilon (x_{\tau s})^3 + \mu(t) \tag{3}$$

where $\mu(t)$ is control function. The expression defined error dynamic of two identical system is obtained as Eq. (4).

$$x_s^{(q)} - x_m^{(q)} = \delta (x_{\tau s} - x_{\tau m}) - \varepsilon \left((x_{\tau s})^3 - (x_{\tau m})^3 \right) + \mu(t) \tag{4}$$

For $\frac{d^q \Delta_\tau}{dt^q} = x_s^{(q)} - x_m^{(q)}$, $\Delta_\tau = x_{\tau s} - x_{\tau m}$ and $\mu(t) = V(t) + \varepsilon \left((x_{\tau s})^3 - (x_{\tau m})^3 \right)$, the error dynamic can be expressed as follow:

$$\Delta_\tau^{(q)} = \delta \Delta_\tau + V(t) \tag{5}$$

If controller is chosen as $V(t) = -K\Delta_\tau$, Eq. (5) transform following form.

$$\Delta_\tau^{(q)} = (\delta - K)\Delta_\tau \tag{6}$$

The characteristic equation of the error dynamic obtained in Eq. (6) is in transcendental fractional order form and its roots give the poles of the error dynamic. For achieving synchronization of the chaotic system, the error between two systems should go to zero. The roots of Eq. (7), the poles of error dynamic, should be left half of s-plane for this.

$$F(s^q) = s^q + (K - \delta) e^{-s\tau} = 0 \tag{7}$$

After the gain value K_c, the error dynamic is critical stable at this parameter, is computed for $s=j\omega$, using the modified Mikhailov stability criterion given in [15-16], range of proportional gain K can be determined. The modified Mikhailov stability function gives in Eq. (8) for this characteristic equation.

$$\psi(s) = \frac{F((j\omega)^q)}{a_0 (j\omega + c)^q}, \qquad c > 0, \tag{8}$$

where a_0 is coefficient of term s^q in Eq. (7). Note that the denominator of Eq. (8) is stable for $c>0$. Equation (8) can be plotted in *Matlab* environment from $\omega=0$ to $\omega=\infty$ for different system parameters and time delay values. This is called the modified Mikhailov stability plot. Then, according to the obtained critical gain, K_c, range of the gain K can be determined.

3. Case Study

In this section, the synchronization of two identical systems given in Eq. (2) and (3) has been carried out by using presented previous section for different initial conditions. For this purpose, the system parameters, which is exhibited chaotic behavior given in [13], have been chosen as q=0.9, $\delta=\varepsilon$=1 and τ=2s. If we substitute $s=j\omega$ into characteristic equation for given system parameters, with help of the Eq. (9), Eq. (7) becomes as presented in Eq. (10), which has real and imagine parts.

$$(j\omega)^q = |\omega|^q e^{jq\frac{\pi}{2}} \tag{9}$$

$$|\omega|^q \cos(q\frac{\pi}{2}) + (K - \delta)\cos(\omega\tau) = 0$$

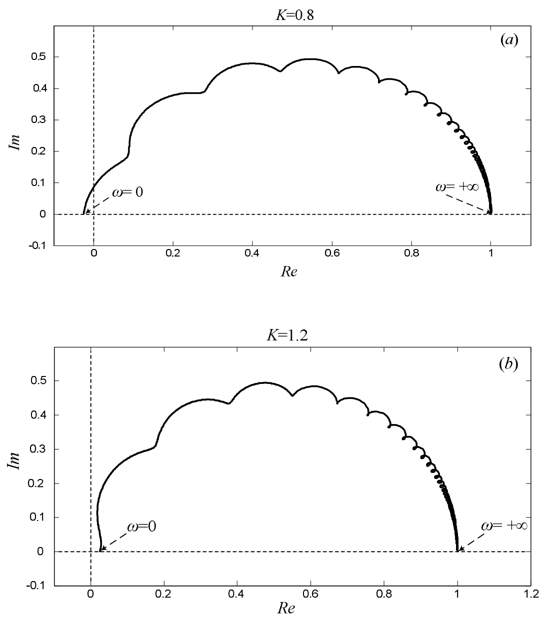
$$\wedge$$

$$|\omega|^q \sin(q\frac{\pi}{2}) + (\delta - K)\sin(\omega\tau) = 0 \tag{10}$$

K_c gain values provide to Eq. (10) as follow:

$$K_{c1} = \delta, \qquad K_{c2} = \delta + \left[\frac{\pi(2-q)}{2\tau} \right]^q \tag{11}$$

K_{c1}=1 and K_{c2}=1.876 have been obtained for the given system parameters. The plot of the expression given in Eq. (8) can be determined the poles of the error dynamic on right side half of s-plane. Fig. 2 shows the modified Mikhailov stability plots (for a_0=1 and c=10) of the error dynamic for different gain values K.



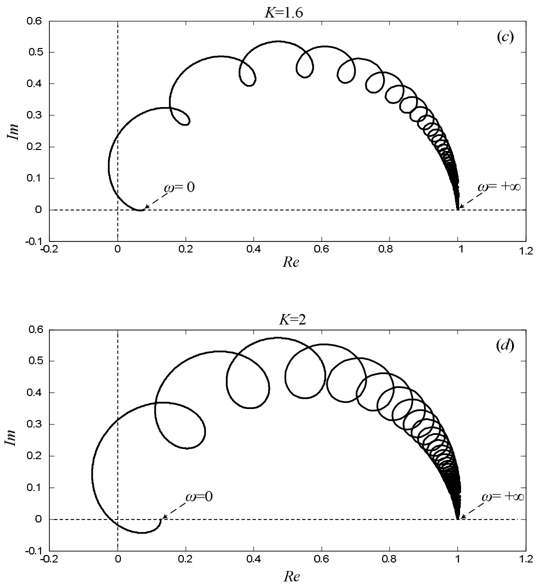
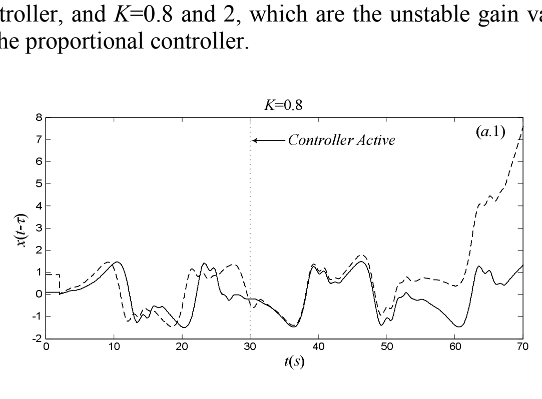


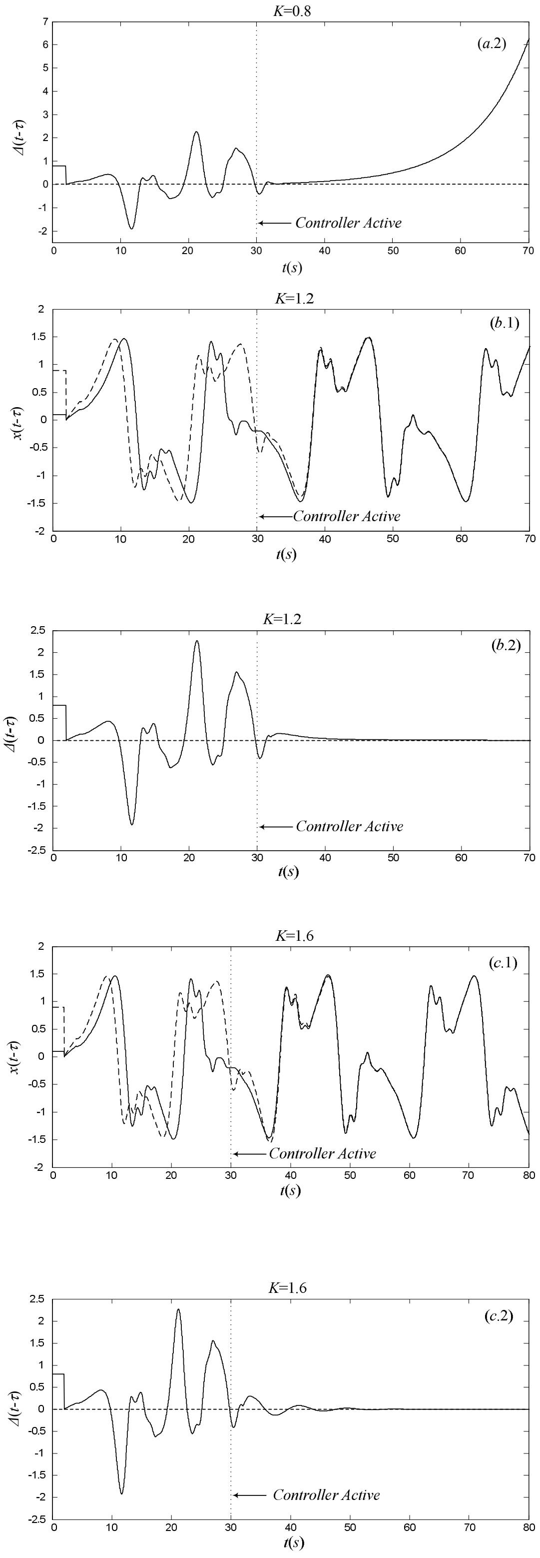
Fig. 2. The modified Mikhailov stability plot for q=0.9, δ=ε=1, τ=2s. ***a)*** K=0.8, ***b)*** K=1.2 , ***c)*** K=1.6 and ***d)*** K=2.

As seen Fig. 2, because modified Mikhailov stability plot don't close the origin, the location of the poles is left side half of s-plane for gain values 1<K<1.876. This means that the Slave system will follow the Master system after the transient response of the error dynamic. On the other hand, because modified Mikhailov stability plot closes the origin, there are the poles on right side half of s-plane for gain values K <1 and K >1.876. This means that the error dynamic is unstable and the synchronization between two systems cannot be achieved.

4. Simulation Results

This section presents the time domain simulations of two identical fractional order time delay chaotic systems using different proportional controllers given previous section on the *Matlab/Simulink*. For this aim, while the initial condition of Master system is chosen x_{0m}=0.1, the initial condition of Slave system is chosen x_{0s}=0.9. The controller provided synchronization has been started up at t=30 s in all of the performed time-domain simulations. Fig. 3 shows the time response of Master and Slave system, and the error for K=1.2 and 1.6, which are the stable gain values of the proportional controller, and K=0.8 and 2, which are the unstable gain values of the proportional controller.





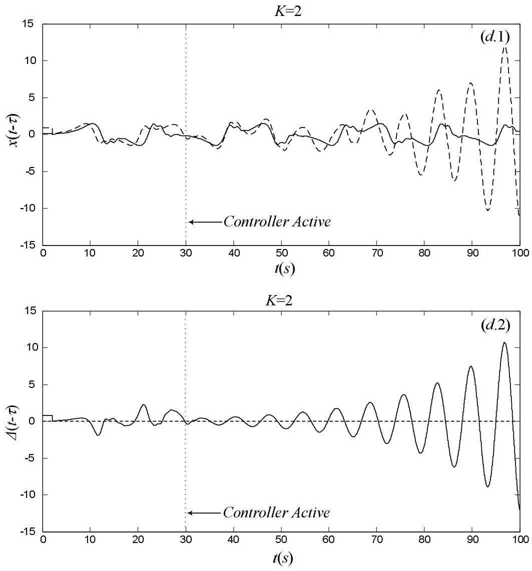


Fig. 3. The time domain simulation results for q=0.9, δ=ε=1 ve τ=2 s output of the Master and Slave systems, and their error ***a)*** K=0.8, ***b)*** K=1.2, ***c)*** K=1.6 and ***d)*** K=2 ((−) *Master* system and (---) *Slave* system).

As seen in Fig. 3, the error dynamic is stable for gain values 1<K<1.876 as predicted previous section. Thus, the synchronization of two chaotic systems can be succeeded for this range of proportional controller gain. Conversely, the error dynamic is unstable for gain values K <1 and K >1.876. Note that the error of the Master-Slave system with proportional gain value K=0.8 increases exponentially. This means that the error dynamic has unstable pole on the real axis of s-plane. For controller gain value K=2, the error increases with oscillation. That is, the error dynamic has complex conjugate unstable poles on the s-plane.

5. Conclusions

This paper has presented the synchronization of two identical fractional order time delay chaotic systems given in Eq. (1) for different initial conditions. The proportional controller has been designed via active control method to provide the synchronization. Using the modified Mikhailov stability criterion, which is graphical-based method, gain values of the proportional controller have been determined. The synchronization can be achieved at the range of K=(1, 1.876) for fractional system order, q=0.9. The designed controller has been verified by the time domain simulation results.

6. References

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