

Mathematical Modeling and Dynamic Analysis of Self-Lift P/O Luo Converter by Means of Signal Flow Graph

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Abstract

Modeling of dc-dc power converters is of great importance to obtain frequency response. Modeling is first step to design compensator circuits for a closed loop system. Computer devices were only able to find dc operating point and to produce diagram boot for circuits with linear elements. Power converters involve diodes and switches and since these elements are non-linear, it is impossible to access dc operating point and diagram boot in such converters. This is while temporal responses can also be simulated. In this article we used current signal graph and Maison operation formula to obtain small signaling model, output control transfer functions (switching), output to input and diagram boot of Luo self boosting converter. Results obtained from simulations in MATLAB software were presented to investigate sustainability and also frequency response.

1. Introduction

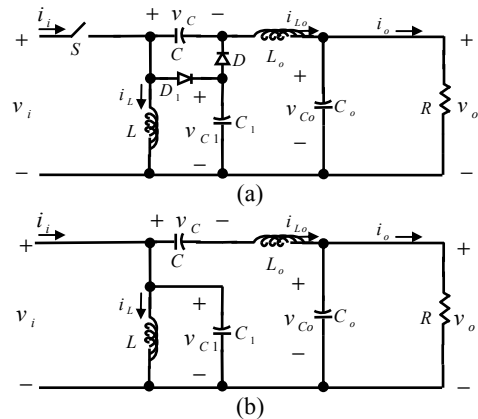
Scholars have presented and developed different mechanisms of dc-dc converters for meeting different needs so far. Existence of an appropriate model of a converter is necessary to design appropriate controller with high efficiency for a dc-dc converter. Moreover using obtained model, it is possible to analyze and investigate sustainability and frequency response of converter. Exploring transfer function is one of simplest ways of analyzing dc-dc converters. Hence, obtaining mathematic model from converter is necessary. Considering that obtaining frequency response of converters to design appropriate controller and compensator circuits requires system modeling and also considering that power converters include non-linear elements such as switches and diodes, then using mathematic model, modeling these converters requires circuits' linearization of the circuits and use of averaging and linearization methods is of great importance in this regard. After linearization of converters, internal model of systems can be explored and doing so, investigation of frequency response will be possible.

The aim of this paper is to obtain small signal model of Luo power electronic converter using signal graph of current method. In this method small signal is obtained using equations averaging and linearization. Using signal graph of current method and Maison operation functions of converting output to input and output control were explored and frequency response were obtained and analyzed in size and boot phase diagrams. The aim of this modeling method is to present method is to present a simple model for such converter. In this article switches and diodes have been considered as ideal and operating mode of converter has been considered as continuous. Traditional methods of small signal averaging and linearization are powerful methods of obtaining basic converters model such as buck and boost, but for converters with high rates such as Sepic, Zeta and Luo, using traditional methods requires dealing with complicated equations. In this article using signal graph of

current method and Maison's operation formula, an appropriate model with simple procedure was obtained through phase to phase analysis. Although converter with switching characteristic has non-linear system, it is possible to analyze it into two linear system (in on and off positions of the switch). These two linear circuits can be displayed by signal graph of current. Current graph of total converter can be obtained by synthesis of two sub graphs and switching branches. Switching branches are the only non-linear elements of converters. So modeling is limited to switching branches. Graphic modeling method was used to study non-linear and dynamic behavior of switching converters. Such converters have different mechanisms and their subsystems are linear. Each subsystem can be displayed by a current graph. Current graph of whole converter is obtained by subsystems' graphs synthesis, using switching branches.

2. Luo Self-Boosting Converter

Luo self-boosting converter power circuit has been shown in Fig. 1(a). Functioning method of this converter can be divided into two operating mode. First state is when switch is turned on and second state is when switch is turned off. Similar circuits for different operating modes of this converter have been shown in Figs. 1(b) and 1(c). This converter involves 3 capacitor and 2 inductors. v_o and v_i are input and output voltages of the converter. v_{C_o} , v_C and v_{C_1} are voltages on C_o , C and C_1 , respectively, and i_{L_o} and i_L are currents passed from L_o and L , respectively. It is supposed that output load is R resistance and D_1 and D diodes and S switch are ideal and converter is at continuous conduct mode state.



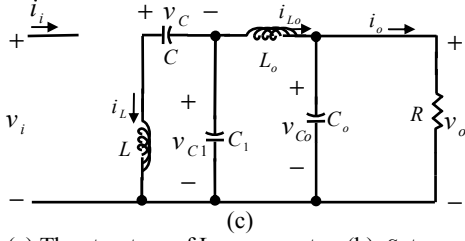


Fig. 1. (a) The structure of Luo converter, (b) S turned on and D turned off, (c) S turned off and D turned on

3. Equations of States of Luo Self-Booster Converter

Equations of states of considered converter were obtained at two operating modes mentioned earlier (when switch is turned on and when switch is turned off), by KVL law and writing equations in loops and nodes. Obtained equations are first degree equations. One advantage of this method is analyzing at higher classes. Since with an increase in circuit degree and number of energy storing elements, all equations will be first degree and this makes analysis simpler, reducing complexity of equations. Considering Fig. 1(b) and considering inductors currents and capacitors voltage as state variables, the equations of capacitors voltages and inductors' current will be obtained as following when S is turned on:

$$L \frac{di_L}{dt} = v_i \quad (1)$$

$$L_o \frac{di_{L_o}}{dt} = v_C - v_o \quad (2)$$

$$(C + C_1) \frac{dv_C}{dt} = -i_{L_o} \quad (3)$$

$$(C + C_1) \frac{dv_{C_1}}{dt} = -i_{L_o} \quad (4)$$

$$C_o \frac{dv_o}{dt} = i_{L_o} - \frac{v_o}{R} \quad (5)$$

According to Fig. 1(c) when S is turned off, equations of state related to converter will be obtained as following:

$$L \frac{di_L}{dt} = v_i - v_C \quad (6)$$

$$L_o \frac{di_{L_o}}{dt} = v_C + v_{C_1} - v_o \quad (7)$$

$$C \frac{dv_C}{dt} = i_L - i_{L_o} \quad (8)$$

$$C_1 \frac{dv_{C_1}}{dt} = -i_{L_o} \quad (9)$$

$$C_o \frac{dv_o}{dt} = i_{L_o} - \frac{v_o}{R} \quad (10)$$

3.1. Averaged Equations

Considering \bar{d} as conductivity coefficient and $1 - \bar{d}$ as S turned off coefficient, the change value of inductors current and capacitors voltage are obtained as equations (11) to (15) as following:

$$\frac{d\bar{i}_L}{dt} = \frac{1 - \bar{d}}{L} v_i - \frac{1 - \bar{d}}{L} v_C \quad (11)$$

$$\frac{d\bar{i}_{L_o}}{dt} = \frac{1}{L_o} \bar{v}_C + \frac{1 - \bar{d}}{L_o} \bar{v}_{C_1} - \frac{1}{L_o} \bar{v}_o \quad (12)$$

$$\frac{d\bar{v}_C}{dt} = -\left(\frac{C + C_1(1 - \bar{d})}{C(C + C_1)}\right) \bar{i}_{L_o} + \frac{1 - \bar{d}}{C} \bar{i}_L \quad (13)$$

$$C_1 \frac{d\bar{v}_{C_1}}{dt} = -\left(\frac{C_1 + C(1 - \bar{d})}{C_1(C + C_1)}\right) \bar{i}_{L_o} \quad (14)$$

$$\frac{d\bar{v}_o}{dt} = \frac{1}{C_o} \bar{i}_{L_o} - \frac{1}{RC_o} \bar{v}_o \quad (15)$$

In these equations variables \bar{x} shows average value which is different from initial variables of x .

3.2. Small Signal Linearization

After obtaining averaged equations, variables of these equations were rewritten and shown with \bar{X} and \tilde{x} , respectively based on variables DC and AC. Considering all variables as $x = \bar{X} + \tilde{x}$, averaged equations are rewritten as following:

$$s(\tilde{i}_L + \bar{I}_L) = \frac{1}{L}(\tilde{v}_i + \bar{V}_i) - \frac{1 - \bar{D} - \tilde{d}}{L}(\tilde{v}_C + \bar{V}_C) \quad (16)$$

$$s(\tilde{i}_{L_o} + \bar{I}_{L_o}) = \frac{1}{L_o}(\tilde{v}_C + \bar{V}_C) - \frac{1}{L_o}(\tilde{v}_o + \bar{V}_o) + \frac{1 - \bar{D} - \tilde{d}}{L_o}(\tilde{v}_{C_1} + \bar{V}_{C_1}) \quad (17)$$

$$s(\tilde{v}_o + \bar{V}_o) = \frac{1}{C_o}(\tilde{i}_{L_o} + \bar{I}_{L_o}) - \frac{1}{RC_o}(\tilde{v}_o + \bar{V}_o) \quad (18)$$

$$s(\tilde{v}_C + \bar{V}_C) = -\left(\frac{C + C_1(1 - \bar{D} - \tilde{d})}{C(C + C_1)}\right)(\tilde{i}_{L_o} + \bar{I}_{L_o}) + \frac{1 - \bar{D} - \tilde{d}}{C}(\tilde{i}_L + \bar{I}_L) \quad (19)$$

$$s(\tilde{v}_{C_1} + \bar{V}_{C_1}) = -\left(\frac{C_1 + C(1 - \bar{D} - \tilde{d})}{C_1(C + C_1)}\right)(\tilde{i}_{L_o} + \bar{I}_{L_o}) \quad (20)$$

In these obtained relations s is considered as Laplas converting operator. Taking this into account that multiplication of two variables DC, DC and two variables AC, AC and two variables AC is zero, inductors' current value and capacitors' voltage are presented as following:

$$s\tilde{i}_L = \frac{1}{L}\tilde{v}_i - \frac{1 - \bar{D}}{L}\tilde{v}_C + \frac{\bar{V}_C}{L}\tilde{d} \quad (21)$$

$$s\tilde{i}_{L_o} = \frac{1}{L_o}\tilde{v}_C - \frac{1}{L_o}\tilde{v}_o + \frac{1 - \bar{D}}{L_o}\tilde{v}_{C_1} - \frac{\bar{V}_{C_1}}{L_o}\tilde{d} \quad (22)$$

$$s\tilde{v}_o = \frac{1}{C_o}\tilde{i}_{L_o} - \frac{1}{RC_o}\tilde{v}_o \quad (23)$$

$$s\tilde{v}_C = -\frac{C + C_1(1 - \bar{D})}{C(C + C_1)}\tilde{i}_{L_o} + \frac{1 - \bar{D}}{C}\tilde{i}_L + \left(\frac{C_1\bar{I}_{L_o}}{C(C + C_1)} - \frac{\bar{I}_L}{C}\right)\tilde{d} \quad (24)$$

$$s\tilde{v}_{C_1} = -\frac{C_1 + C(1 - \bar{D})}{C_1(C + C_1)}\tilde{i}_{L_o} + \frac{C\bar{I}_{L_o}}{C_1(C + C_1)}\tilde{d} \quad (25)$$

and discussed in two sections of forward paths from \tilde{v}_i to \tilde{v}_o and forward paths from \tilde{d} to \tilde{v}_o .

4.2.1. Forward Paths From \tilde{v}_i to \tilde{v}_o

Considering signal graph of current, there is a path from \tilde{v}_i to \tilde{v}_o . This path and its operation are shown in relation 48 as following:

$$P_i: \tilde{v}_i \rightarrow \tilde{s}\tilde{i}_L \rightarrow \tilde{i}_L \rightarrow \tilde{s}\tilde{v}_C \rightarrow \tilde{v}_C \rightarrow \tilde{s}\tilde{i}_{Lo} \rightarrow \tilde{i}_{Lo} \rightarrow \tilde{s}\tilde{v}_o \rightarrow \tilde{v}_o \rightarrow P_i = \frac{(1-\bar{D})}{LL_oCC_oS^4} \quad (48)$$

4.2.2. Forward Paths From \tilde{d} to \tilde{v}_o

Considering signal graph of current, there is two paths from \tilde{d} to \tilde{v}_o . These paths and their operations are shown in relation (49) and (50) as following:

$$P_{d1}: \tilde{d} \rightarrow \tilde{s}\tilde{i}_L \rightarrow \tilde{i}_L \rightarrow \tilde{s}\tilde{v}_C \rightarrow \tilde{v}_C \rightarrow \tilde{s}\tilde{i}_{Lo} \rightarrow \tilde{i}_{Lo} \rightarrow \tilde{s}\tilde{v}_o \rightarrow \tilde{v}_o \rightarrow P_{d1} = \frac{\bar{V}_i}{LL_oCC_oS^4} \quad (49)$$

$$P_{d2}: \tilde{d} \rightarrow \tilde{s}\tilde{i}_{Lo} \rightarrow \tilde{i}_{Lo} \rightarrow \tilde{s}\tilde{v}_o \rightarrow \tilde{v}_o \rightarrow P_{d2} = \frac{\bar{V}_i}{L_oC_o(1-\bar{D})^2S^2} \quad (50)$$

5. Transfer Function of Luo Converter

Transfer functions of Luo converter was obtained by Maison's operation formula. Considering form of Miosen's operation formula, replacing relations $P_i\Delta_i$ in Maison's operation formula and after simplifying them, the transfer function will be obtained as following:

$$\frac{\tilde{v}_o}{\tilde{v}_i} = \sum \frac{P_i\Delta_i}{\Delta} = \frac{A_1s + A_0}{D_5s^5 + D_4s^4 + D_3s^3 + D_2s^2 + D_1s + D_0} \quad (51)$$

Considering form of Miosen's operation formula, $P_d\Delta_d$, after eliminating P_{d1} path, $\Delta_1 = 1$. By replacing relations in Maison's operation formula and after simplifying them transfers function of $\frac{\tilde{v}_o}{\tilde{d}}$ will be obtained as following:

$$\frac{\tilde{v}_o}{\tilde{d}} = \sum \frac{P_d\Delta_d}{\Delta} = \frac{B_3s^3 + B_2s^2 + B_1s + B_0}{D_5s^5 + D_4s^4 + D_3s^3 + D_2s^2 + D_1s + D_0} \quad (52)$$

$A_0, A_1, B_0, B_1, B_2, B_3, D_0, D_1, D_2, D_3, D_4,$ and D_5 values have been presented in (53)–(64) as following:

$$A_0 = 0 \quad (53)$$

$$A_1 = \frac{(1-\bar{D})}{LL_oCC_o} \quad (54)$$

$$B_0 = 0 \quad (55)$$

$$B_1 = \frac{2\bar{V}_i}{LL_oCC_o} \quad (56)$$

$$B_2 = 0 \quad (57)$$

$$B_3 = \frac{\bar{V}_i}{L_oC_o(1-\bar{D})^2} \quad (58)$$

$$D_0 = \frac{(C_1 + C(1-\bar{D}))(1-\bar{D})^3}{RLL_oCC_1C_o(C+C_1)} \quad (59)$$

$$D_1 = \frac{(C_1 + C(1-\bar{D}))(1-\bar{D})^3}{LL_oCC_1(C+C_1)} + \frac{(1-\bar{D})^2}{LL_oCC_o} \quad (60)$$

$$D_2 = \frac{(1-\bar{D})^2}{RLCC_o} + \frac{C+C_1(1-\bar{D})}{RL_oCC_o(C+C_1)} + \frac{(C_1 + C(1-\bar{D}))(1-\bar{D})}{RL_oC_1C_o(C+C_1)} \quad (61)$$

$$D_3 = \frac{(1-\bar{D})^2}{LC} + \frac{C+C_1(1-\bar{D})}{L_oC(C+C_1)} + \frac{(C_1 + C(1-\bar{D}))(1-\bar{D})}{L_oC_1(C+C_1)} + \frac{1}{L_oC_o} \quad (62)$$

$$D_4 = \frac{1}{RC_o} \quad (63)$$

$$D_5 = 1 \quad (64)$$

6. Simulation and Analysis of Frequency Response

To investigate sustainability and frequency response, a Luo converter with following elements value was considered:

$$L=L_o=1mH, C_o=C_1=C=20mF, R=40\Omega, v_i=20v, D=0.5$$

Roots locus of the converter can be seen in Fig. 3 and magnitude and phase of bode diagram curves have been shown in Fig.4.

Considering roots locus curves and data obtained from $\frac{\tilde{v}_o}{\tilde{v}_i}$

transfer function, this converter has a real pole and 4 mixed conjunctive poles at left side imaginary axis and a zero on coordination destination. Considering values of magnitude and phase of bode diagram, when phase diagram is -180 degree, then the value of magnitude $-5dB$. Since this value is less than $0dB$, so this transfer function will be sustainable. When value diagram is $0dB$, phase diagram will shows 46° . Phase limit equal to difference of this number is -180 which is obtained 226° . Roots locus for transfer function of Luo converter can be seen in Fig. 5 and data and obtained from zeros

and poles of $\frac{\tilde{v}_o}{\tilde{d}}$ transfer function, magnitude and phase of bode diagram can be seen in Fig.6. Considering roots locus and data

obtained from $\frac{\tilde{v}_o}{\tilde{d}}$ transfer function, this converter has a real

pole and 4 mixed conjunctive poles at left side of imaginary axis and a zero on coordination destination and two conjunctive poles zeros on imaginary axis. Considering roots locus and magnitude and phase of bode diagram, transfer function is sustainable in ratio of all operation values, because the place is usually tend to imaginary axis and operation limit is infinite and considering time boot diagram which is $0dB$ in value, the phase is -175° and phase limit which is obtained from discrepancy of this number with -180° is 5° .

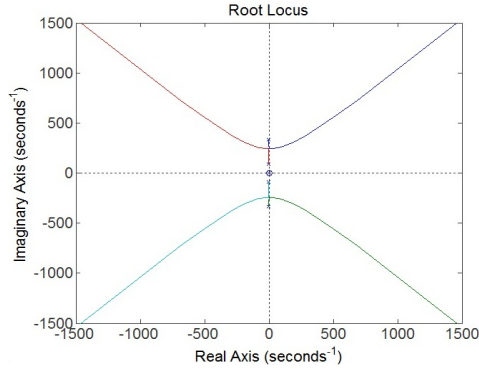


Fig. 3. Roots locus of transfer function $\frac{\tilde{v}_o}{\tilde{v}_i}$

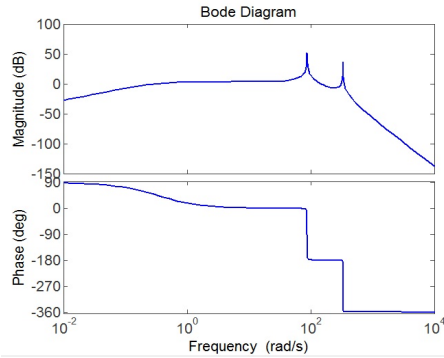


Fig. 4. Roots diagram of transfer function $\frac{\tilde{v}_o}{\tilde{v}_i}$

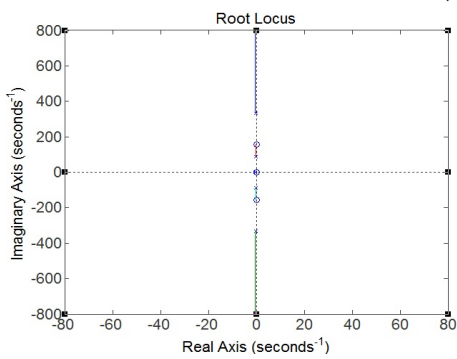


Fig. 5. Roots locus of transfer function $\frac{\tilde{v}_o}{d}$

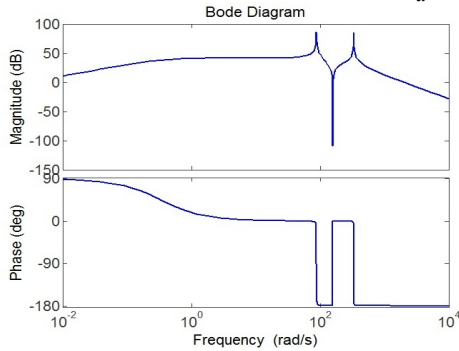


Fig. 6. Roots diagram of transfer function $\frac{\tilde{v}_o}{d}$

Table 1. Information data of poles under transfer function $\frac{\tilde{v}_o}{\tilde{v}_i}$

Pole	corner's frequency	zero	corner's frequency (rad/s)
$-0.186 \pm 88i$	88	0	0
$-0.269 \pm 333i$	333	0	0
-0.341	0.341	0	0

	(rad/s)		
$-0.186 \pm 88i$	88	0	0
$-0.269 \pm 333i$	333	0	0
-0.341	0.341	0	0

Table 2. Information data of poles under transfer function $\frac{\tilde{v}_o}{d}$

Pole	corner's frequency (rad/s)	zero	corner's frequency (rad/s)
$-0.186 \pm 88i$	88	0	0
$-0.269 \pm 333i$	333	0	0
-0.341	0.341	$\pm 158i$	158

7. Conclusion

In this article mathematic modeling of Luo converter was carried out using signal graph of current and Mason's operation formula. Moreover averaging method along with small signal linearization was used and steps of obtaining and exploring final transfer function from initial equations or the same circuit equations were presented step by step. As it was shown using signal graph of current and Mason's operation formula is a simple method for modeling converters with high degree. This method provides the situation for managing converters with complex equations. For this aim, the transfer function for controlling to output and input to output was obtained. It can be seen that results of operation and zero place, poles, boot diagrams and also possibility of analyzing frequency response can be easily accessed from proposed model.

8. References

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